



## รายงานการวิจัยฉบับสมบูรณ์

การออกแบบและสร้างระบบสื่อสารไร้สายแบบปลอดภัยด้วยเทคนิคการเคลือบสัญญาณอลวนบน  
สัญญาณเสียงและภาครับแบบซิงโครไนซ์ปรับตัว

Design and Development of Secure Wireless Communication System  
Using Chaotic Masking Technique and Adaptive-Synchronization Receiver

เสนอต่อฝ่ายวิจัยและบริการวิชาการ สถาบันเทคโนโลยีไทย-ญี่ปุ่น

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## 1. Introduction and Backgrounds

Chaos is a phenomenon that occurs in nonlinear dynamical systems. Such deterministic dynamical systems have their origin in Newtonian physics, leading to the systematic development in differential calculus. The “physical laws of nature” in the Newtonian sense propose to model all phenomena by deterministic laws describing the flow of system states [1]. Mathematicians and physicists have sought to understand the world in terms of these deterministic laws, and due to the lack of powerful computation devices, searched for closed form solutions for deterministic dynamical systems. Unfortunately, the existence of closed form solutions is essentially limited to the case of linear dynamical systems. This did, however, not limit the optimism of visionaries conjecturing that, with the knowledge of all “physical laws of nature”, predictions about the however remote future of dynamical systems are feasible. In order to cope with the incompleteness of the deterministic description available about a system, the concept of randomness has been introduced to capture all behavior counters the concept of predictability. While a number of important ideas refer to Cardano, Bernoulli and, in particular Gauss and Fermat, there are mainly the results of Kolmogorov in the 1930's [2] that influenced modern probability theory. Another manifestation of randomness comes from statistics, in which by the very nature of the problem, a complete description of the underlying dynamical system is not available.

For the dynamical [3] systems approach, it was Poincare' observed that determinism does not necessarily lead to predictability without limits (though the consequences were not completely understood at his time). Indeed, Poincare' described the intrinsic nature of what has later been called “chaotic behavior”, their sensitive dependence on initial conditions. An example is the computer processing evolution of a rather complex dynamical system. Despite the initial optimism it has been found unpredictable over a longer time period. However, this unpredictability need not be linked to complex systems. For the very example of the weather, Loren found deterministic unpredictability even in a very simplified third order module. a small perturbation of the state of the system would very soon lead to a macroscopically different evolution of the system

In the past years, synchronization of chaotic systems problem has received a great deal of attention among scientists in various fields. As it is well known, the study of the synchronization problem for nonlinear systems has been very important from the nonlinear science point of view, in particular, the applications to biology, medicine, cryptography and secure data transmission. In general, synchronization research has been focused on two areas. The first one relates to the employment of state observers, where the main applications pertain to the synchronization of nonlinear oscillators. The second one is the

use of control laws, which allows achieving the synchronization between nonlinear oscillators, with different structure and order. Of particular interest is the connection between the observers for nonlinear systems and chaos synchronization, which is also known as master-slave configuration. Therefore, chaos synchronization problem can be posed as an observer design procedure, where the coupling signal is viewed as output and the slave system is regarded as observer. The general idea for transmitting information via chaotic systems is that, an information signal is embedded in the transmitter system which produces a chaotic signal, the information signal is recovered when the transmitter and the receiver are identical. Since Pecora and Carroll's [1-3] observation on the possibility of synchronizing two chaotic systems, several synchronization schemes have been developed. Synchronization can be classified into mutual synchronization and master slave synchronization.

There are many applications to chaotic communication and chaotic network synchronization. The techniques of chaotic communication can be divided into three categories (1) Chaos masking; the information signal is added directly to the transmitter. (2) Chaos modulation; it is based on the master-slave synchronization, where the information signal is injected into the transmitter as a nonlinear filter. (3) Chaos shift keying; the information signal is supposed to be binary, and it is mapped into the transmitter and the receiver. In these three cases, the information signal can be recovered by a receiver if the transmitter and the receiver are synchronized. In order to reach synchronization, the receiver should be a replica of the transmitter.

Security is an important for communication. Where of two entities are communicating in a way not susceptible to eavesdropping or interception. This is known secure communication. This includes means by which people can share information with varying degrees of certainty that third parties cannot intercept. There have been considerable interests in chaotic communications over the past several years. Synchronization caused great interest in science and technology workers. The research of the application of chaos synchronization in secret communication is the most competitive research fields in recent years. Most of the secure communication system requirements of the signal modulation, as far as possible the regular and has strong anti-interference ability to interpret.

The synchronization methods have been proposed in a number of the related theories and experiment results. The existing problems encountered includes relatively high decoding errors, slow decoding of receiver processing signals, large size of circuit implementation, which may lead to the application to the communication. Various unseen result were still in which theoretical forms which have not been practically utilized in recent years. Therefore, the chaotic jerk oscillator may be a potential

alternative the size and simple synchronization. Feedback reconstruction may lead to an error reduction and also perform a faster synchronization.

## 2. Objectives

- 2.1 To design and implement compact cost-effective chaotic circuits.
- 2.2 To design and implement the chaotic-masking secure communication system.

## 3. Research Scopes

- 3.1 Study the dynamical system including chaos theory, nonlinear analysis through the mathematical model such as dynamic equation, time-scaling model, Eigen value, Eigen-Vector, Jacobian-Matrix and also Stability analysis. Study chaotic indicators such as attractor, time-domain, Poincare' section, Bifurcation, Lyapunov diagram and Kaplan-York dimension [1-3]. The research will also enhance the model of chaotic function by generalizing the form of the chaotic function equation.
- 3.2 Implement chaotic circuit in secure communication systems for simulation and design including chaotic circuit by generalizing the form of the chaotic function equation.
- 3.3 Implement the chaotic-masking secure walky-talky communication system.

## 4. Liturature Reviews

### 4.1 Chaos Theory and Dynamical System

Chaos is a short word describe a behavior of dynamical [2] systems which appears closely to random, however, the chaotic systems can be rewritten through the set of nonlinear equation systems. Such the chaotic systems appear normally in natural environments. Chaos and randomness are generally due to the chaotic characteristic still confused with the random behavior. Chaos can occur only in nonlinear systems and characterized by a breakdown of predictability known as sensitive dependence on initial conditions which is the most important distinguishing feature of chaos. This implies that even though chaotic systems are deterministic, even the smallest difference in initial state can cause a dramatically difference in the final state. Long term predictability of chaotic systems is impossible since

all numerical calculations have a finite non-zero error which will diverge over time and the predictions unreliable. The chaotic behavior contain three majors properties , (1) chaos can occur only in deterministic nonlinear dynamical systems,(2) Chaotic behavior looks complicated and irregular but has an infinite number of unstable periodic patterns embedded in the system and (3) chaotic behavior is sensitive to initial conditions

## 4.2 Dynamic Systems

A dynamical system is one whose state changes in time. If the changes are determined by specific rules, rather than being random. The system is deterministic; otherwise it is stochastic. The changes can occur at discrete time steps or continuously. This book will be concerned with continuous-time, deterministic, dynamical systems since they arguably best approximate the real world. This view represents the prejudice of most physical scientists, but it is also the case that chaos is relatively too easy to achieve in discrete-time systems, and hence it is less of a challenge to find elegant examples of chaos in such systems, and those that are found have less apparent relevance to the natural world. Also, discrete-time systems have already been extensively explored, in part because they are more computationally tractable. Stochastic systems mimic many of the features of chaos, but they are not chaotic because chaos is a property of deterministic systems. Furthermore, introducing randomness into a dynamical model is a way of admitting ignorance of the underlying process and obtaining plausible behavior without a deep understanding of its cause.

## 4.3 Summary of Related Chaos from Dynamic Systems

Table 1 shows literature review of chaos jerk function, which are initially reviewed as Chaos theory. Six particularly related chaos theory for secure communication have previously been proposed by Ken Kiers and Dory Schmidt(2003), VinodPatidar and K KSud(2005), GuoboXie and et al.(2008), Ljubiša M. Kocić and Sonja Gegovska-Zajkova (2009) and BunchaMunmuangsae and et al. (2011)



Table 1 Summary of existing chaos jerk functions.

| Authors                                     | Year | Title  |
|---|------|--|
| Ken Kiers and Dory Schmidt                  | 2003 | Precision measurements of a simple chaotic circuit                                       |
| Vinod Patidar and K K Sud                   | 2005 | Bifurcation and chaos in simple jerk dynamic   |
| Guobao Xie and et al                        | 2008 | Generation of multidirectional multi-scroll attractors under the third-order Jerk system |
| Ljubiša M. Kocić and Sonja Gegovska-Zajkova | 2009 | On a Jerk dynamical systems  |
| Buncha unmuangsae and et al                 | 2011 | Generalization of the simplest autonomous chaotic system                                 |

First, Ken Kiers and Dory Schmidt [5], describe a simple nonlinear electrical circuit that can be used to study chaotic phenomena. The circuit employs simple electronic elements such as diodes, resistors, and operational amplifiers, and is easy to construct. A novel feature of the circuit is its use of an almost ideal nonlinear element, which is straightforward to model theoretically and leads to excellent agreement between experiment and theory. The circuit contains three successive inverting integrators with outputs at the nodes labeled  $V_2$ ,  $V_1$ , and  $x$ , as well as a summing amplifier with its output at  $V_3$ . If we use Kirchhoff's rules at nodes a-d.

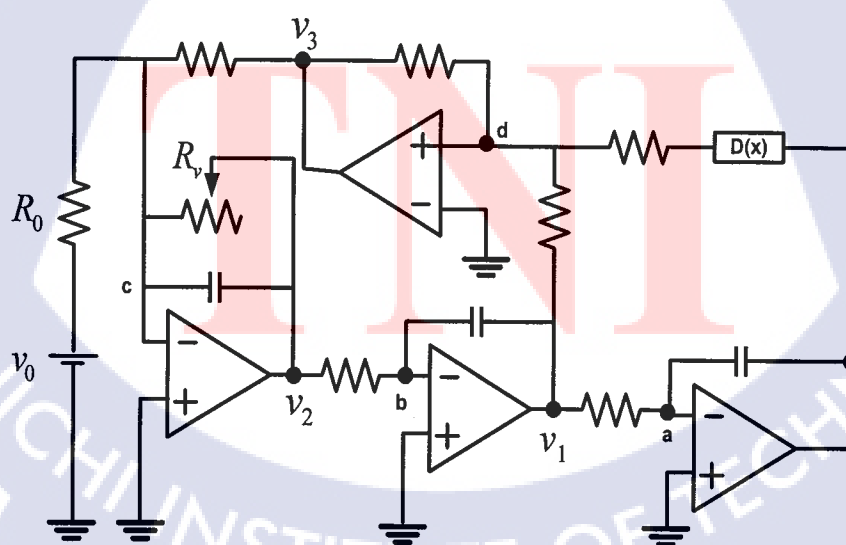


Figure 1 Schematic diagram of the circuit presented by Ken Kiers and Dory Schmidt.

Second, Vinod Patidar and K K Sud have derived the recursive proportional feedback algorithm and shown that it can be used to control chaotic oscillations in the Kiers, Schmidt, and Sprott electronic circuit. Control is well within the uncertainty of the target fixed point. The values of the coefficients used in the recursive proportional feedback algorithm were calculated from experimentally measured values of the output voltage of the circuit during precontrol measurements. Recursive proportional feedback is suitable for highly dissipative systems, of which the KSS circuit is an example. Simple proportional feedback is also suitable for some highly dissipative systems, but cannot be used for the KSS circuit because the movement of the system's chaotic attractor through phase space depends on both the current and previous perturbations.

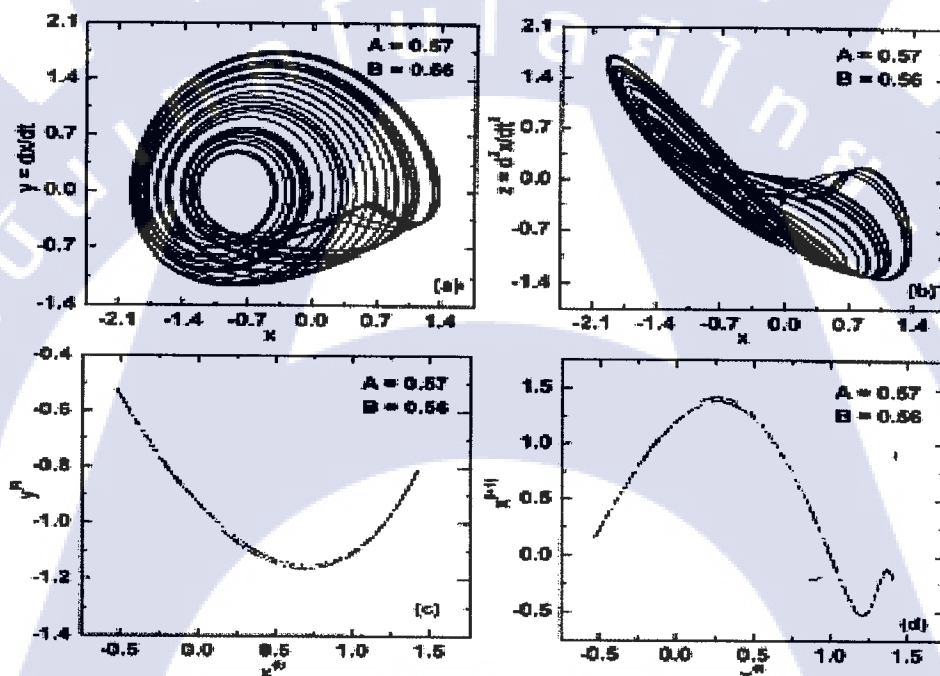


Figure 2. Strange attractor of the jerk dynamical system having quadratic on-linearity.

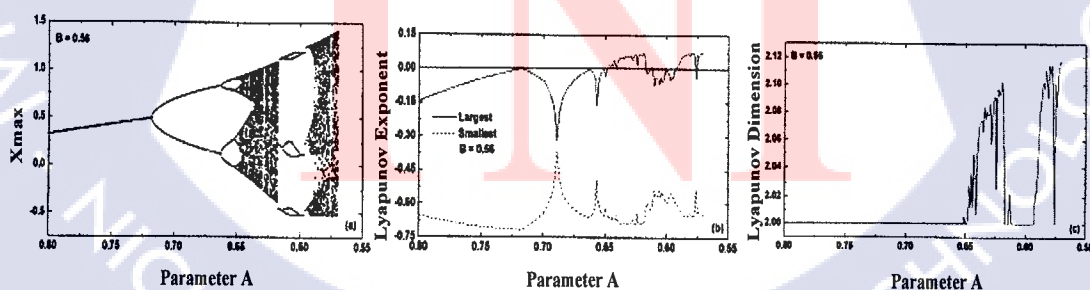


Figure 3 Behavior of the jerk dynamical system having quadratic non-linearity for a fixed value of parameter  $B = 0.56$

Third, GuoboXie and et al. [7], In this paper, An approach for generating multi directional grid chaotic attractors from a third-order Jerk system is proposed via constructing a series of staircase functions, including two-directional and three- directional multi-scroll chaotic attractors. Its dynamical behaviors are investigated by means of theoretical analysis as well as numerical simulation.

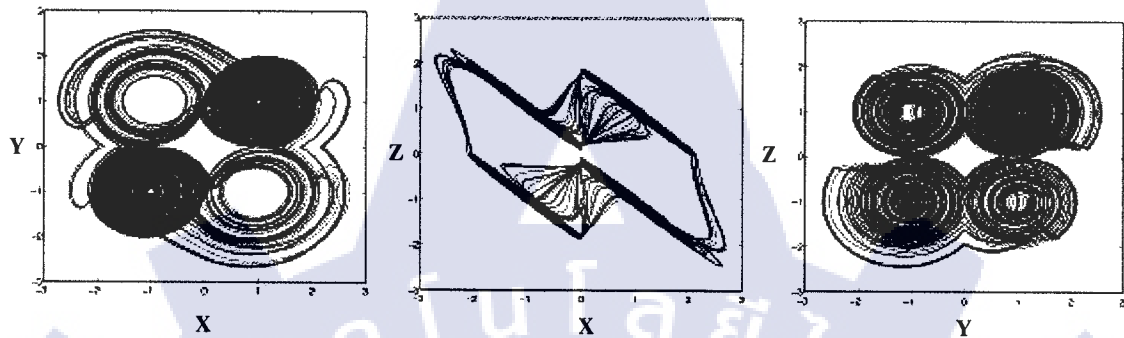


Figure 4. 2x2x2 grid-scroll chaotic attractors presented by GuoboXie and et al..

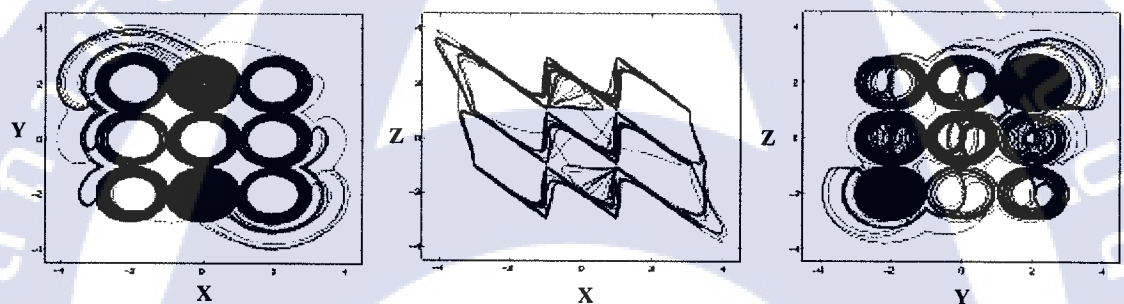


Figure 5. 3x3x3 grid-scroll chaotic attractors presented by GuoboXie and et al..

Forth, Ljubiša M. Kocić and Sonja Gegovska-Zajkova [8], chaotic systems of J.C. Sprott emanated from electric circuits turn to be attractive examples of week chaos the only form of chaos that eventually might be acceptable in sensible applications like automatic control or robotics. Here, two modifications of a 3D dynamic flow, known as jerk dynamical system of J.C. Sprott are considered. The left-semi quadratic system  $\ddot{x} = -Ax + g_k(\dot{x}) - x$ . The system preserves chaotic regime of the original Sprott setting  $\ddot{x} = -Ax + \dot{x}^2 - x$  for lesser values of  $A$  and bigger slope values  $k$ . The choice  $A = 1.3$  and  $k = 1$  produces recognizable phase diagrams, given in two projections in Fig. 2.24. The  $x(t)$  diagram and the DFT confirms chaotic element, and the Lyapunov coefficient is  $\lambda_1 = 0.0338091$ . One of the simplest dynamic flows that still exhibits chaotic behavior is Sprott's jerky system, given by equations  $\ddot{x} = -Ax + \dot{x}^2 - x$ , where  $\psi(\xi) = \xi^2$ . This system "works" on the "edge" of chaos, which is evident from its small first Lyapunov exponent ( $\lambda_1 = 0.0551$ ). Tracing for simpler function  $\phi$  that yet supplies chaotic dynamics, Sprott and Linz tried



with  $\psi(\xi) = |\xi|$ , the continuous function that represents a kind of piecewise linear approximation of quadratic function. In this case, no chaos has been detected. The present note deals with the “hybrid” case embodied in two semi-quadratic functions, the left- and the right one, see Fig.2.24 for the graphs. Surprisingly, the left semi-quadratic function  $g_k(\xi)$  produces chaos for some values of the larger part’s variable slope  $k$ , and the corresponding value of  $A$ , while the symmetric, right semi-quadratic function  $h_k(\xi)$  leads only to a non-chaotic dynamics.

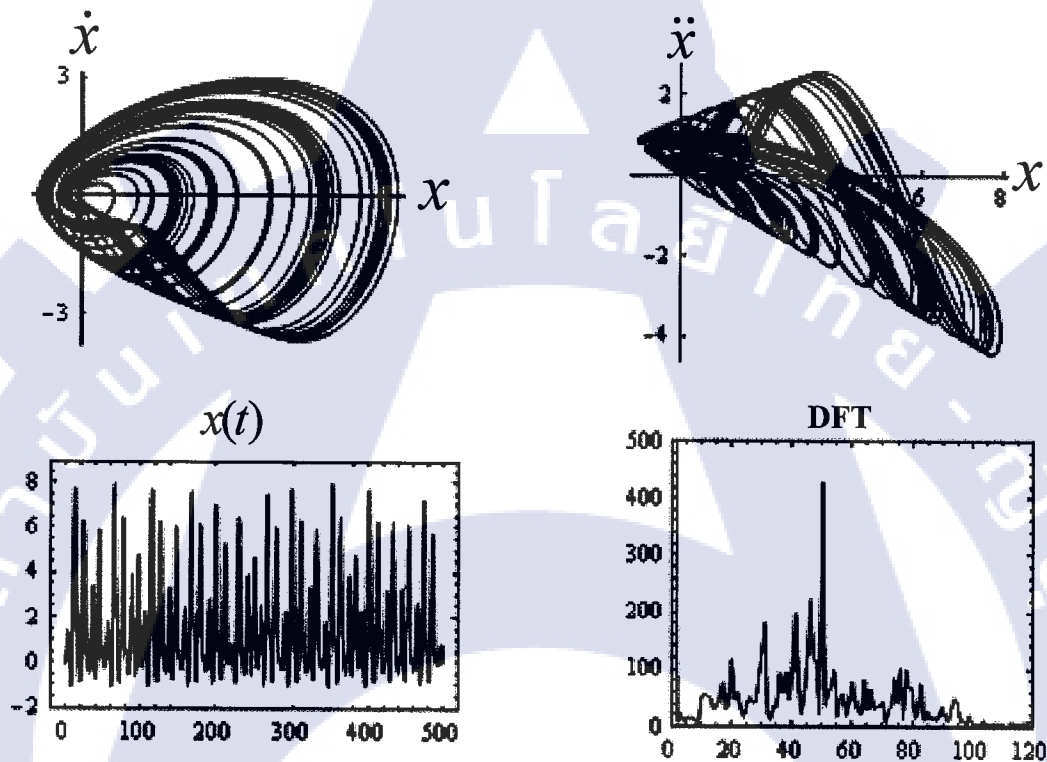
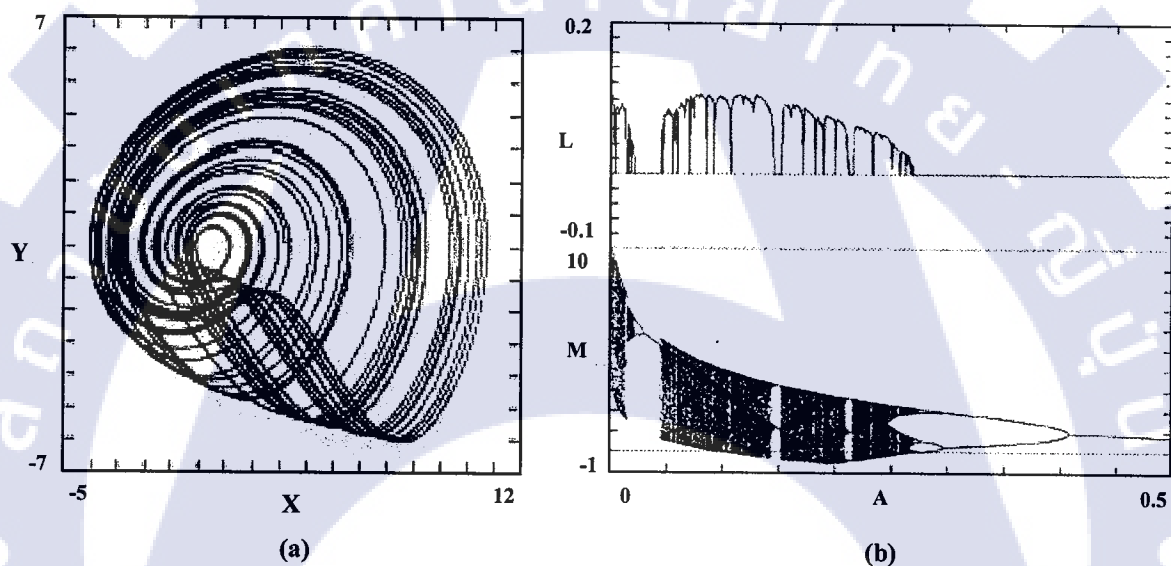


Figure 6 The  $x(t)$  diagram and the DFT confirms chaotic element with  $A = 1.3$ ;  $k = 1$ ;  $\lambda_1 = 0.0338091$

presented by Ljubiša M. Kocić and Sonja Gegovska-Zajkova[7].

Last, Buncha Munmuangsae and et al. [9], an extensive numerical search of jerk systems of the form  $\ddot{\ddot{x}} + \dot{x} + x = f(x)$  revealed many cases with chaotic solutions in addition to the one with  $f(x) = \pm x^2$  that has long been known. Particularly simple is the piecewise-linear case with  $f(x) = \alpha(1-x)$  for  $x \geq 1$  and zero otherwise, which produces chaos even in the limit of  $(\alpha \rightarrow \infty)$  the dynamics in this limit can be calculated exactly, leading to a two-dimensional map. Such nonlinearity suggests an elegant electronic circuit implementation using a single diode. This raises the question of whether there are other simple chaotic systems of the form and with  $A = 0.1$ , is particularly interesting. It has the curious feature of having chaotic solutions in the limit of  $A \rightarrow 0$  as is evident from its largest Lyapunov exponent and

bifurcation diagram (the local maxima of  $x$ ) shown in Figure 2.25, which shows a period-doubling route to chaos. The attractor grows in size as  $A \rightarrow 0$  since a larger  $x^*$  is required to achieve the same nonlinearity as  $A$  decreases. The equilibrium point for this case has Eigenvalue  $= .15193, 0.2596 \pm 0.7686i$ , which satisfies the Shilnikov condition since the absolute value of the real Eigenvalue is greater than the absolute value of the real part of the complex Eigenvalue, providing a proof of chaos. For this value of  $A$ , the largest Lyapunov exponent is near its maximum with a Lyapunov exponent spectrum of  $(0.1016, 0, .1.1016)$  and a Kaplan–Yorke dimension of  $DKY = 2.0922$ . In conclusion, several simple chaotic systems of the form  $\ddot{x} + \dot{x} = f(x)$  have been studied. They have similar maximum values of their largest Lyapunov exponents and corresponding Kaplan–Yorke dimensions. Furthermore, all cases have  $F^{\wedge}'(x) > -1$  with spiralsaddles of index 2. Particularly simple is the piecewise-linear case with  $f(x^*) = \alpha(1-x^*)$  for



**Figure 7** (a) Attractor form equation and (b) The largest Lyapunov exponent and bifurcation diagram of equation for  $f(x) = -A \exp(x)$  with  $0 < A < 0.5$  presented by Buncha Munmuangsae and et al.

#### 4.4 Summary of Related Chaotic Communication Systems

Table 2 show application chaotic systems for communication, which is initially reviewed as purpose chaotic system for communication. Nine mainly linked chaos application chaotic system for communication have previously been proposed by Shihua Chen and et al.(2003), Pehlivan and Y. Uyaroglu(2007), Dandan Zhao and et al.(2008), GaoBingkun and et al.(2009), Said SADOUDI and Mohamed Salah Azzaz(2009), Ihsan Pehlivan and et al. (2010), Jiejing Liu and Yanli Zhang(2011) and Jing Pan and Qun Ding(2011)

**Table 2** Summary of applications of chaotic system for communications.

| Author                        | Year | Title  |
|-------------------------------|------|--|
| Shihua Chen and et al.        | 2003 | Adaptive synchronization of uncertain Rossler hyper chaotic system based on parameter identification   |
| Pehlivan and Y. Uyaroglu      | 2007 | Simplified chaotic diffusion less Lorenz attractor and its application to secure communication systems |
| Gao Bingkun and et al.        | 2009 | The application research of Hyper chaos encryption in security communications                          |
| Said SADOUDI and Mohamed S.A. | 2009 | Hardware Implementation of the Rössler Chaotic System for Securing Chaotic Communication               |
| Ihsan Pehlivan and et al.     | 2009 | Design and simulations of the Arneodo attractor's chaotic oscillator and signal masking circuit        |

First, Shihua Chen and et al. presents an approach of adaptive synchronization and parameters identification of uncertain Rössler hyper chaotic system is proposed. The suggested tool proves to be globally and asymptotically stable by means of Lyapunov method. With this new and effective method, parameters identification and synchronization of Rössler hyper chaotic with all the system parameters unknown, can be achieved simultaneously. Theoretical proof and numerical simulation demonstrate the effectiveness and feasibility of the proposed technique. Rössler hyper chaotic system was provided by Rössler in describing dynamics of some hypothetical chemical reaction and is a first example of hyper chaotic system with two positive Lyapunov exponents.

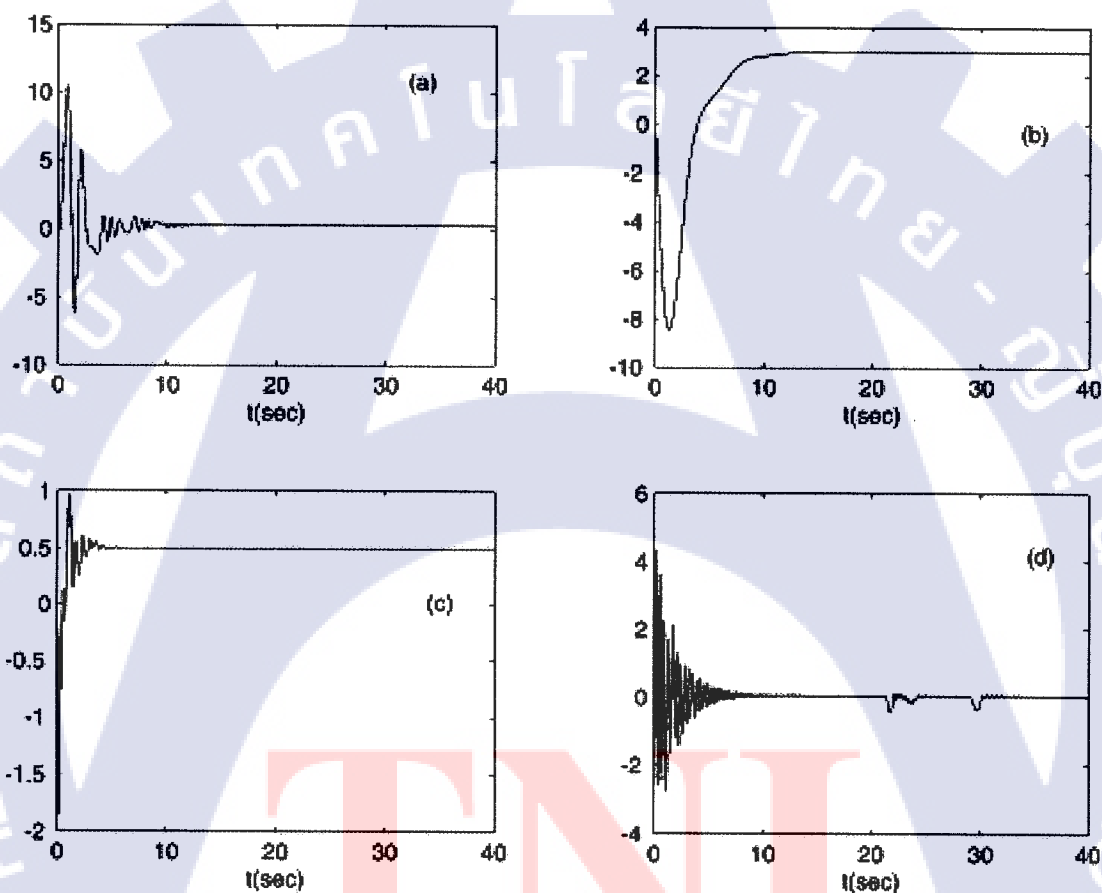


Figure 8 Graphs of parameters identification results presented by Shihua Chen and et al..

Second, Pehlivan and Y. Uyaroglu, this show Diffusion less Lorentz equations a simplified one-parameter version of the well-known Lorentz model. Also, it was attained in the limit of high Rayleigh and Prandtl numbers, physically corresponding to diffusion less convection. A simplified, one-parameter



version of the Lorentz model called diffusion less Lorentz is proposed, which is suitable for chaotic synchronization and masking communication circuits using Matlab/Simulink and Spice programmers. It is also suitable for a real electronic experimental circuit.

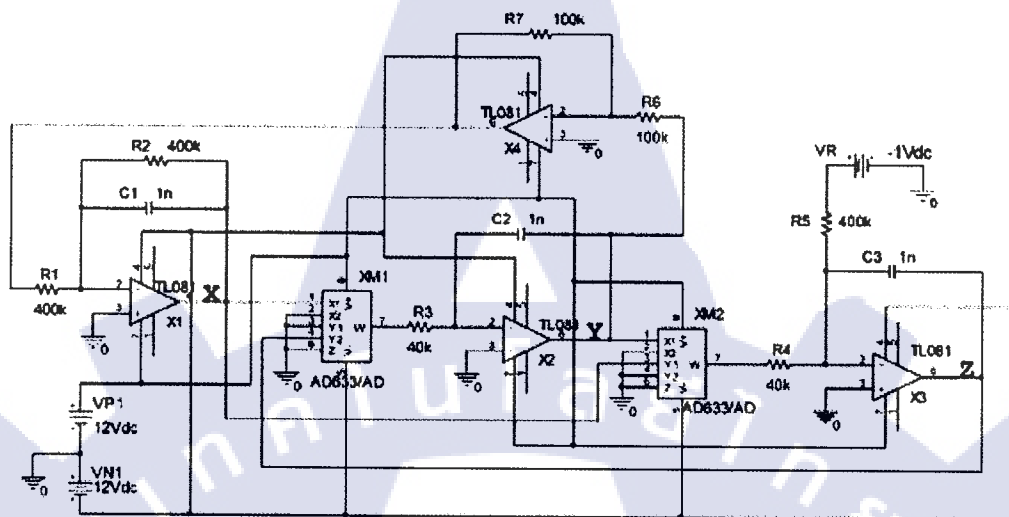


Figure 9 Spice circuit presented by Pehlivan and Y. Uyaroglu.

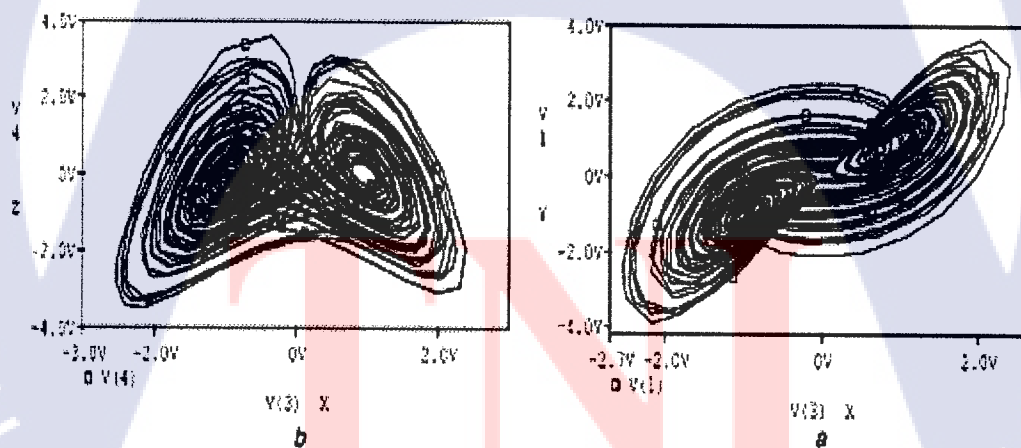


Figure 10 Spice simulation results (a) x, y phase portrait (b) x, z phase portrait presented by Pehlivan and Y. Uyaroglu.

Third, GaoBingkun, Li Wenchao and Hu Yue [12], against the nature of hyperchaos dynamic system, a modified hyper-chaotic sequence encryption algorithm was given. This method used the dynamic system of TNC Hyper chaos. And proved to have the effective ability of exhaustive attack and anti-nonlinear reorganization; Used Sub-NY Quist sampling interval to increase the key space, this method has the ability of anti-nonlinear reorganization attack; realized the algorithm's encryption and decryption by using MATLAB simulation platform and got some satisfactory results which make out that the arithmetic is faster and easier implementation by software, had a large key space and so on. cryptogram designs flexible and has a large design space. It can enhance security, provide possibilities of a solution to improve the limited effect which result in short cycles, provide a guarantee for improving anti-exhaustive attacks. It can provide a large key space. TNC circuit equation has a large number of system variables and parameters which can be used as the seed key for the sequence cryptosystem. The algorithm not only has a large key space but also can improve the security.

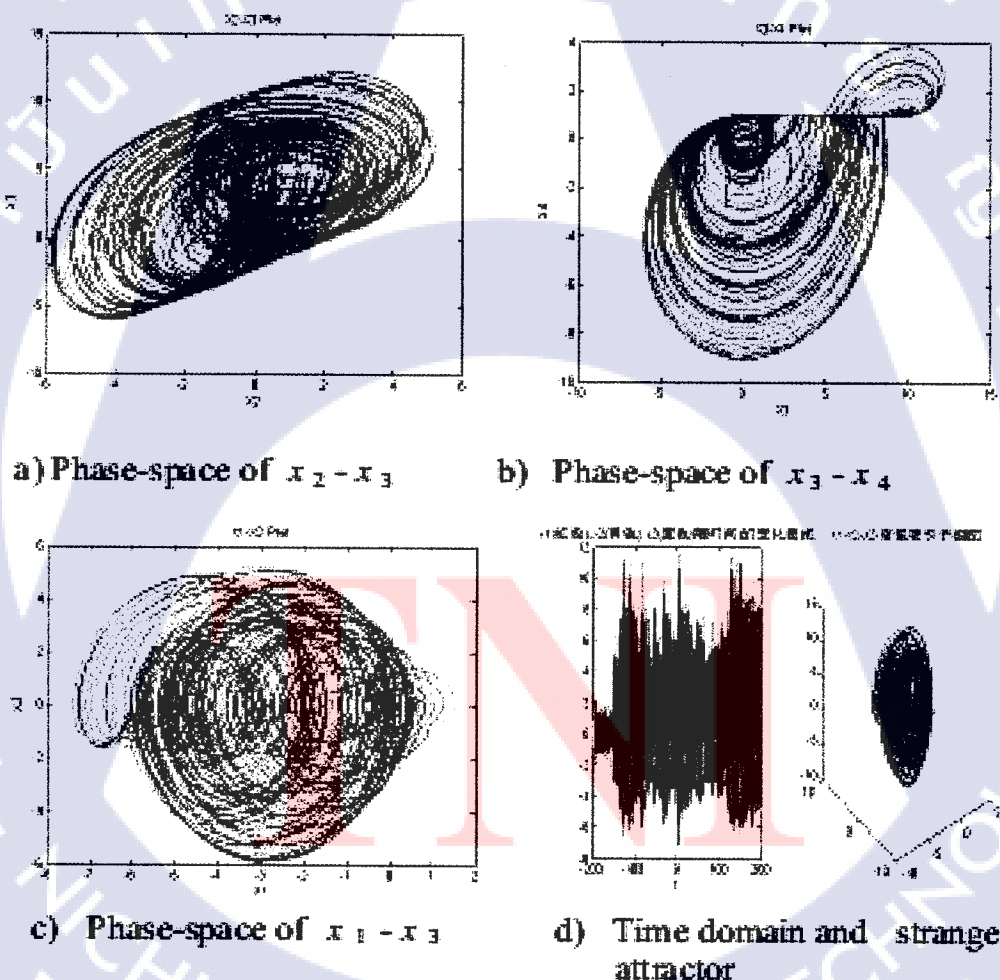
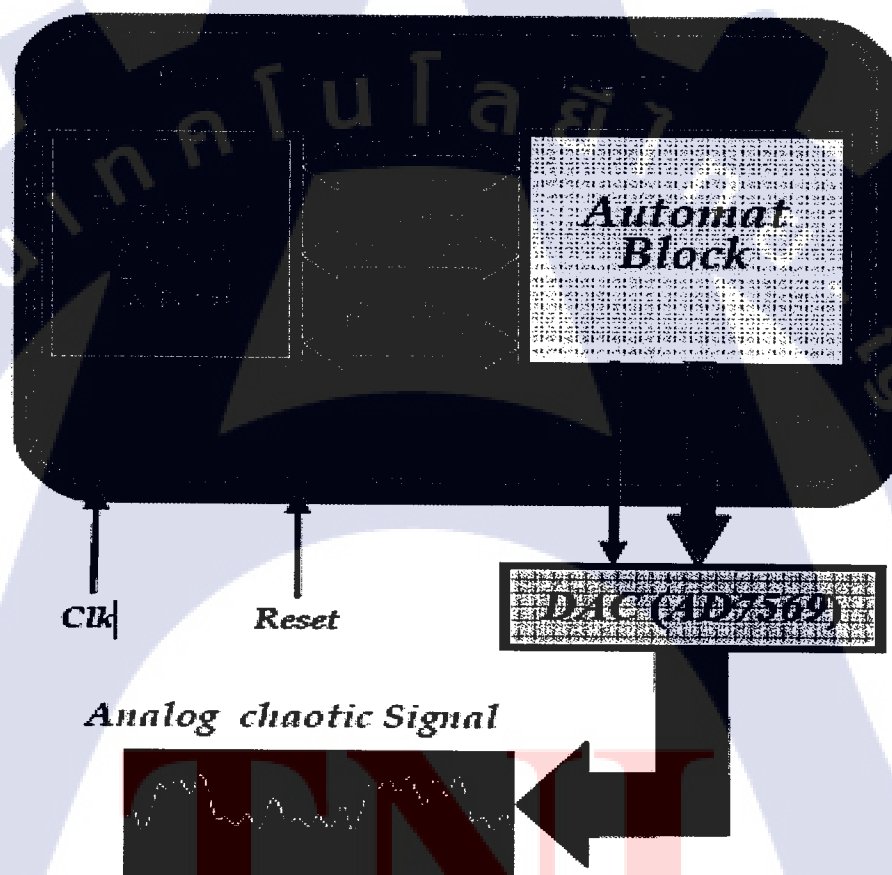


Figure 11 Attractors of TNC Hyper chaos circuit presented by GaoBingkun, Li Wenchao and Hu Yue.

Forth, Said Sadoudi and Mohamed Salah Azzaz presents a real-time implementation of the Rössler chaotic system in a Field Programmable Gate Array(FPGA) is presented. At first, we use directly the VHDL language for the hardware description of the system, contrary to some previous works where the Xilinx system generator of MATLAB-Simulink used to generate the VHDL code. Then, after a step of optimization, to reduce the resources of the circuit target Virtex-II xcv1000-4fg456, we implement the chaotic system on FPGA. The real-time chaotic signals obtained at the output of the FPGA are then compared with those obtained by MATLAB and Models simulation, in order to validate our results. However, the goal of this work is to introduce this chaotic system in an eventual secure digital chaotic communication system. O. E. Rössler introduced his equations system in 1976.



**Figure 12** Scheme of the digital implementation of Rössler chaotic system on FPGA presented by Said Sadoudi and Mohamed Salah Azzaz.

Last, IhsanPehlivan, YılmazUyaroglu and M. Ali Yalcinve Selçuk Coskun presents synchronizing two coupled ratchet Josephson junctions subjected to a quasiperiodic field is achieved. In the limit of weak perturbation of irrational frequencies equal to the square root of the transcendental number and for

small damping parameters, phase locking occurs as the coupling between both junctions is increased. It turns out that the transition from non-synchronous to synchronous chaotic state does not involve attractors appearing and disappearing. The undertaken symmetry analysis of the system demonstrates the suppression of the massive phase fluctuations as the coupling rises, allowing chaos synchronization between both junctions to take place. The calculations also reveal the persistence of the synchronous state for high coupling strengths, taking into consideration the symmetry particularity of the external drive and potential.

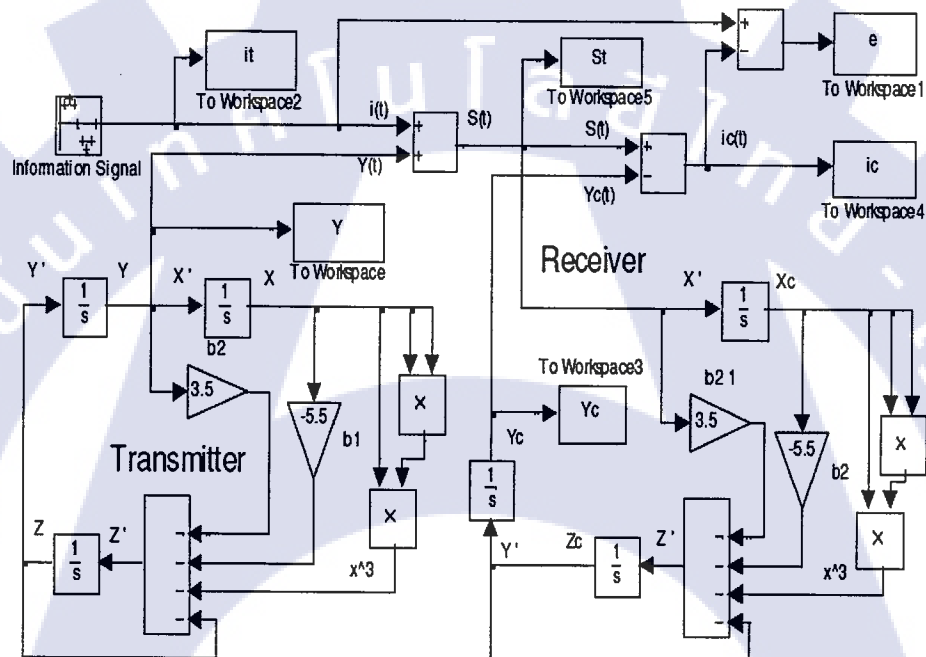
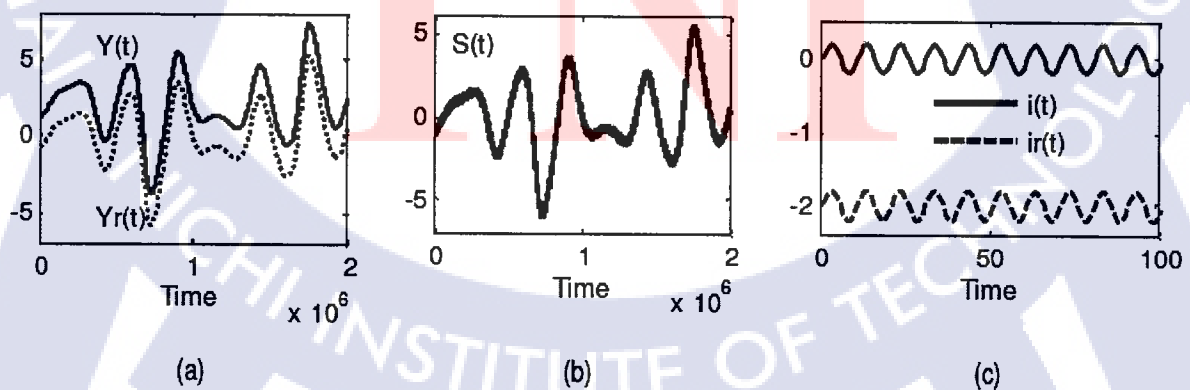


Figure 13 Simulink modeling of chaotic masking communication circuit of the Arneodo Attractor presented by Ihsan Pehlivan and M. Ali Yalcinve Selçuk Coskun





**Figure 14** Simulink outputs of Masking Communication Scheme of Arneodo Attractor (a) Drive ( $y$ ) and response ( $y_r$ ) system chaotic signals vs. Time, (b) Transmitted signal  $S(t) = y(t) + i(t)$ , c) Information  $i(t)$  and retrieved  $i_r(t)$  signals (sinus signal) has 0.2V amplitude and frequency 10 KHz presented by Ihsan Pehlivan and M. Ali Yalcinve Selçuk Coskun.

## 5. Proposed Chaotic Flow and Its Results

### 5.1 Proposed Rössler Attractor using Diode Equation

In 1963, Edward Lorenz [21] encountered sensitively dependent initial conditions of an atmospheric convection model while performing numerical simulations leading to the discovery of the Lorenz system with seven-terms in three-dimensional ordinary differential equations and two quadratic nonlinearities. In 1976, Rössler [24] proposed a chaotic system with seven terms and a single quadratic nonlinearity, which is algebraically simpler than Lorenz system. In addition, a single folded-band attractor of Rössler system is topologically simpler than a two-scroll Lorenz attractor. Such Lorenz and Rössler systems have consequently led to considerable research interests in searching for new chaotic systems with fewer terms in ODEs [5-10] or more complex attractor topology.

Several chaotic systems with fewer than seven terms and two quadratic nonlinearities continuously been reported as variants in Lorenz system family. Complex three-scroll and four-scroll attractors based on Lorenz system have also been suggested through the use of three or more quadratic nonlinearities. On the other hand, simple chaotic systems with a single nonlinearity similar to Rössler system are rarely found. In fact, Rössler himself had proposed another system with six terms and a single quadratic nonlinearity in 1979 [30-35]. In 1994, Sprott [15-19] found fourteen cases with six terms and a single quadratic nonlinearity through an intensive numerical computer search. Recently, many simple systems have been proposed in simple Jerk equations with single quadratic or non-quadratic

nonlinearities. Despite the fact that these simple Jerk chaotic systems with a single nonlinearity potentially resemble the single folded-band Rössler attractor, the Kaplan-York dimension ( $D_{KY}$ ) as a measure of complexity is somewhat lower than the original Rössler attractor that possesses the greatest value of  $D_{KY} = 2.1587$ . This leads to a question of whether the original Rössler system in dynamic forms can be simplified into fewer terms with simple nonlinearity, or modified for more complex attractor. No simplifications of Rössler system has never been found so far.

## 5.2 Dynamical Properties

Based on the Rössler system proposed in 1979, the first and the second equations, i.e.  $\dot{x} = -y - z$  and  $\dot{y} = x + ay$ , initiate a normal band of the attractor through an outward spiral motion into the x-y phase plane. Nonlinear interactions between x and z variables in the third equation, i.e.  $\dot{z} = b + z(x - c)$ , form an additional folded band to the overall attractor. It is noticeable that the folded band in Rössler attractor rises and returns exponentially in z-dimension especially for positive values of x variable under the flows. This aspect implies that the third equation may be modified through the use of an exponential nonlinearity. Therefore, a new chaotic system is therefore presented in three-dimensional autonomous ODEs expressed in a general form as

$$\begin{aligned}\dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= -z + bF(x)\end{aligned}\tag{1}$$

where  $(x, y, z) \in \mathbb{R}^3$  are dynamical variables,  $(a, b) \in \mathbb{R}^+$  are system parameters, and  $F(x)$  is a nonlinear function required for chaos. Two particularly simple cases of the nonlinear function  $F(x)$  are presented using exponential functions. In other words,

$$F_1(x) = e^x\tag{2}$$

### 5.3 Numerical analysis

The existence of attractor can be described by the divergence of flows. For a dissipative chaotic system,  $p < 0$  and therefore  $a$  is limited into the region  $0 < a < 1$ . The exponential rate of contraction is  $dV/dt = e^{(a-1)t}$  and hence a volume element  $V_0$  is contracted in time  $t$  by the flows into a volume element  $V_0 e^{-t}$ . Each volume containing the system trajectories shrinks to zero as time  $t$  approaches  $+\infty$ . All system orbits will be confined to a specific limit set of zero volume, and the asymptotic motion converges onto an attractor. It can be concluded that the existence of attractors is constant and independent to the nonlinear term  $bF(x)$ .

### 5.4 Bifurcations, Lyapunov Exponents, and Kaplan-Yorke Dimension

Numerical simulations have been performed in MATLAB using the initial condition of  $(x_0, y_0, z_0) = (1, 0, 1)$ . In fact, the initial condition is not crucial, and can be selected from any point that lies in the basin of attractor. In order to find the control parameter  $a$  that offers the maximum values of chaoticity and complexity, Figure 4.1 shows the bifurcation diagram of the peak of  $z$  ( $z_{\max}$ ) versus the parameter  $b$ . It is seen in Figure 4.1 that the system exhibits a period-doubling route to chaos. In addition, Figure 4.2 shows the plots of the positive LE versus the parameter  $b$ . The chaoticity is a measure of the greatest LE, which is the average rate of growth of the distance between two nearby initial conditions that grows exponentially in time when averaged along the trajectory, leading to long-term unpredictability property. The Lyapunov exponents can be employed for the estimation of the rate of entropy production and the fractal dimension commonly known as Kaplan-Yorke dimension  $D_{KY}$ , i.e.

$$D_{KY} = j + \frac{1}{|LE_{j+1}|} \sum_{i=1}^j LE_i = k + \frac{LE_1 + LE_2}{|LE_3|} \quad (3)$$

where  $k$  is a non-integer constant, and typically equals to 2 for three-dimensional chaotic systems.

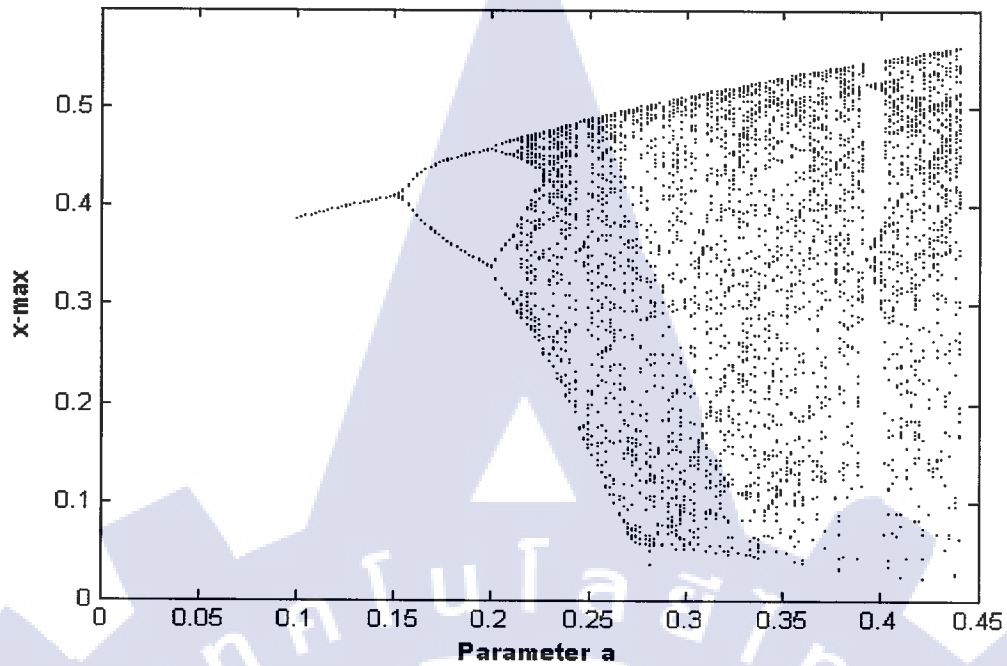


Figure 15 Bifurcation diagram fixed  $b=0.0007$ .

### 5.5 Numerical Equilibria and Eigenvalue

The Jacobian of the system is

$$\begin{aligned} 0 &= -y - z \\ 0 &= x + ay \\ 0 &= -z + be^x \end{aligned} \quad (4)$$

where  $x$ ,  $y$  and  $z$  are the state variables and  $a$ , bare positive real constants. The system displays a typical chaotic attractor when  $a = 0.2$  and  $b=0.00045$ . The new system has equilibrium points  $(0, 0, 0)$

$$J = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ bF' & 0 & -1 \end{bmatrix} \quad (5)$$

Applying the equilibrium point  $P$  into this Jacobian matrix and analyzing  $|\lambda I - J| = 0$  reveal a resulting characteristic polynomial as follows:



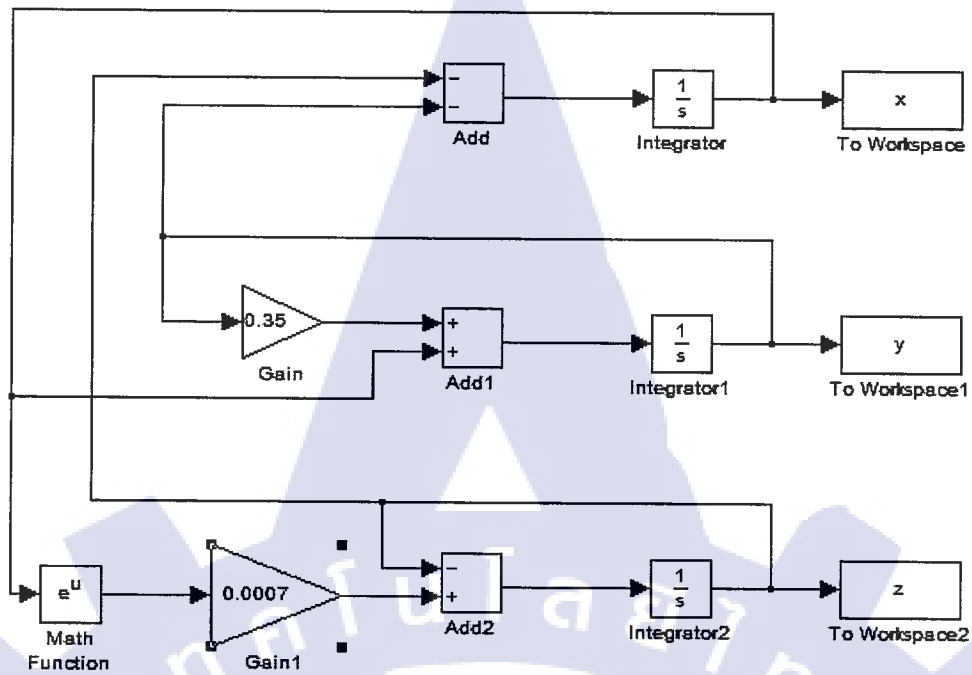


Figure 16 Chaotic scheme Rössler attractor using Matlab Simulink.

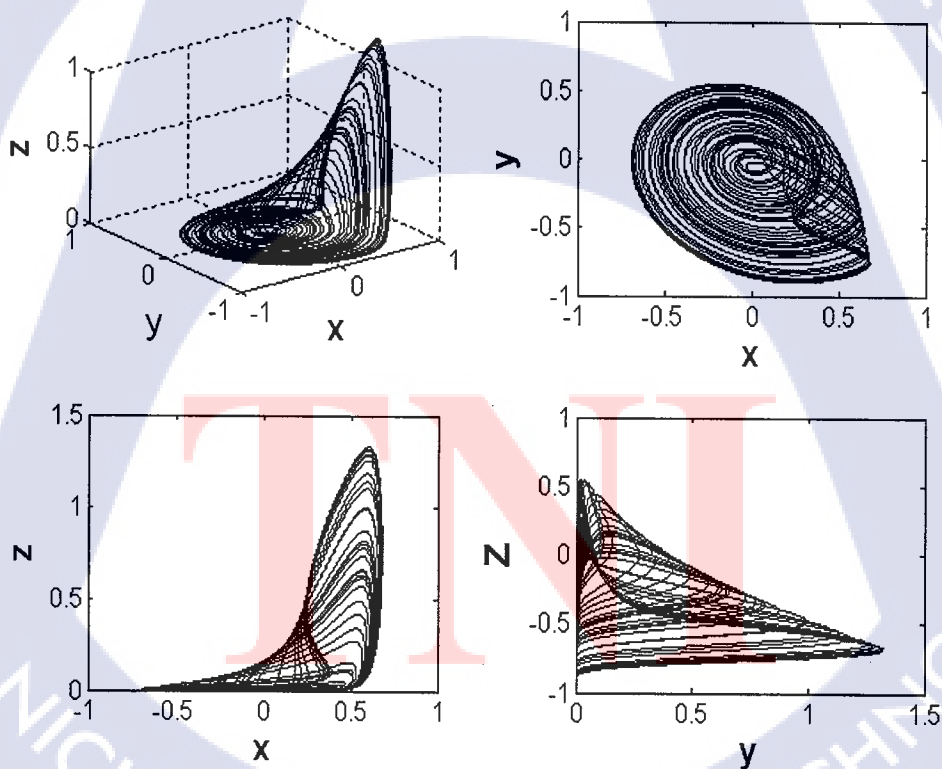


Figure 17 Simulation Phase portraits with  $F_1(x) = e^x$  at  $a=0.30$  and  $b=0.0007$ ,  $LEs = (0.0638, 0, -0.8641)$ ,  
 $D_{KY} = 2.0738$ .

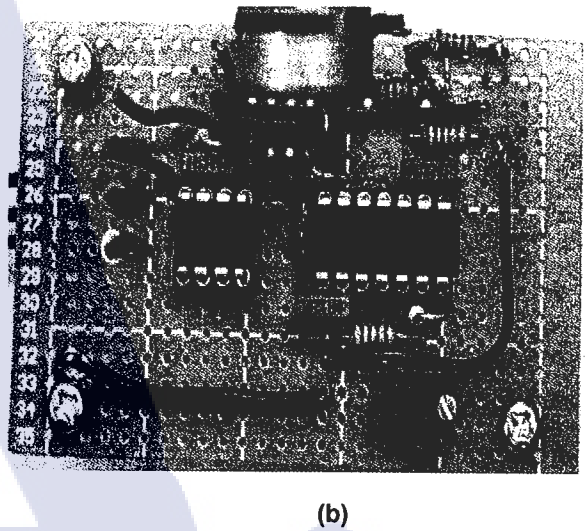
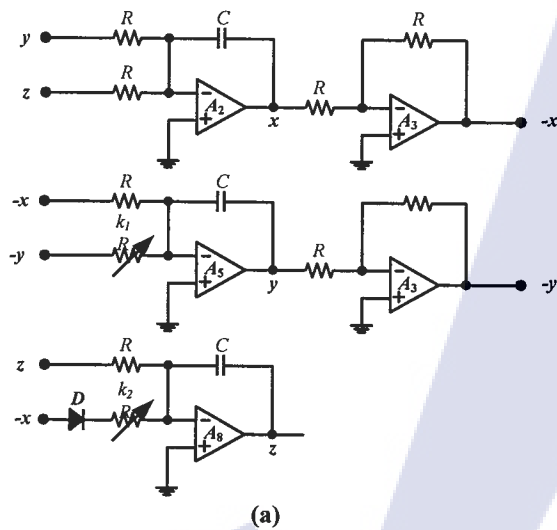
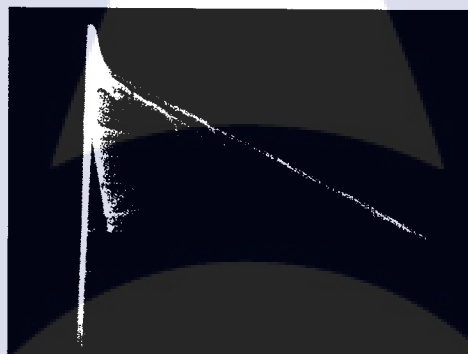
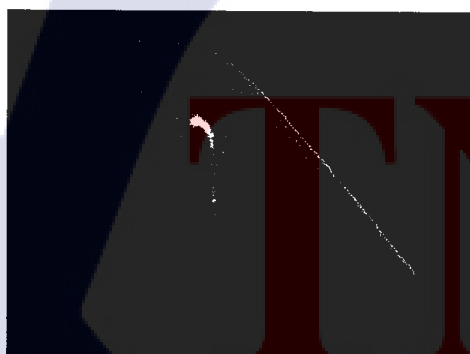


Figure 18 Chaotic circuits (a) design chaotic circuit and (b) real chaotic circuit



x-z phase portrait



x-y phase portrait



y-z phase portrait

Figure 19 Result chaotic attractor of real electronics circuit.

$$\lambda^3 + (1-a)\lambda^2 + (bF' - a + 1)\lambda + (abF' - 1) = 0 \quad (6)$$

According to the Routh–Hurwitz stability criterion, the system (1) is unstable when  $F' < (1 + (1-a)^2)/(b-2ab)$ . Note that dynamic behaviors depend on two parameters  $a$  and  $b$ , and can be characterized completely by the plot of parameter space without redundancy. For all particular values of  $a$  and  $b$  in the subsequent numerical analyses, the resulting eigenvalue  $\lambda_1$  is a positive real number and  $\lambda_2$  and  $\lambda_3$  are a pair of complex conjugate with positive real parts, indicating that the equilibrium points are all saddle focus points. Fig.18 shows the circuit schematic for implementing real electronics circuit form eq. 4.1. We use TL081 op-amps, diode 2N4148,  $R = 10\text{k}\Omega$ ,  $C = 1\text{nF}$  and potentiometer.  $k_1 = 44\text{ k}\Omega$  and  $k_2 = 0.7\text{ k}\Omega$ . The circuit is supplied  $\pm 9\text{V}$ . As show Figure 19 shows the result experimental chaotic attractor from oscilloscope. Figure 4.10 shows the comparison between simulation and experiment when  $b = 0.0007$  and  $R_b = 0.7\text{ k}\Omega$ .

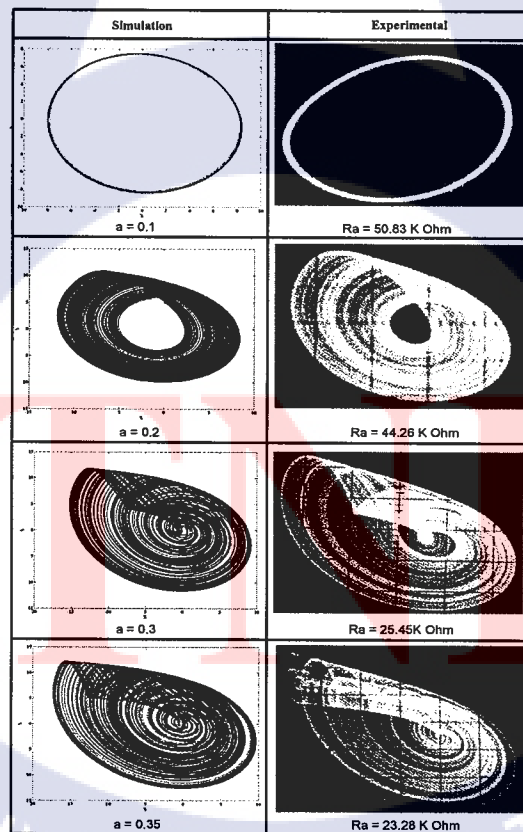


Figure 20 Comparison between simulation and experiment when  $b = 0.0007$  and  $R_b = 0.7\text{ k}\Omega$ .

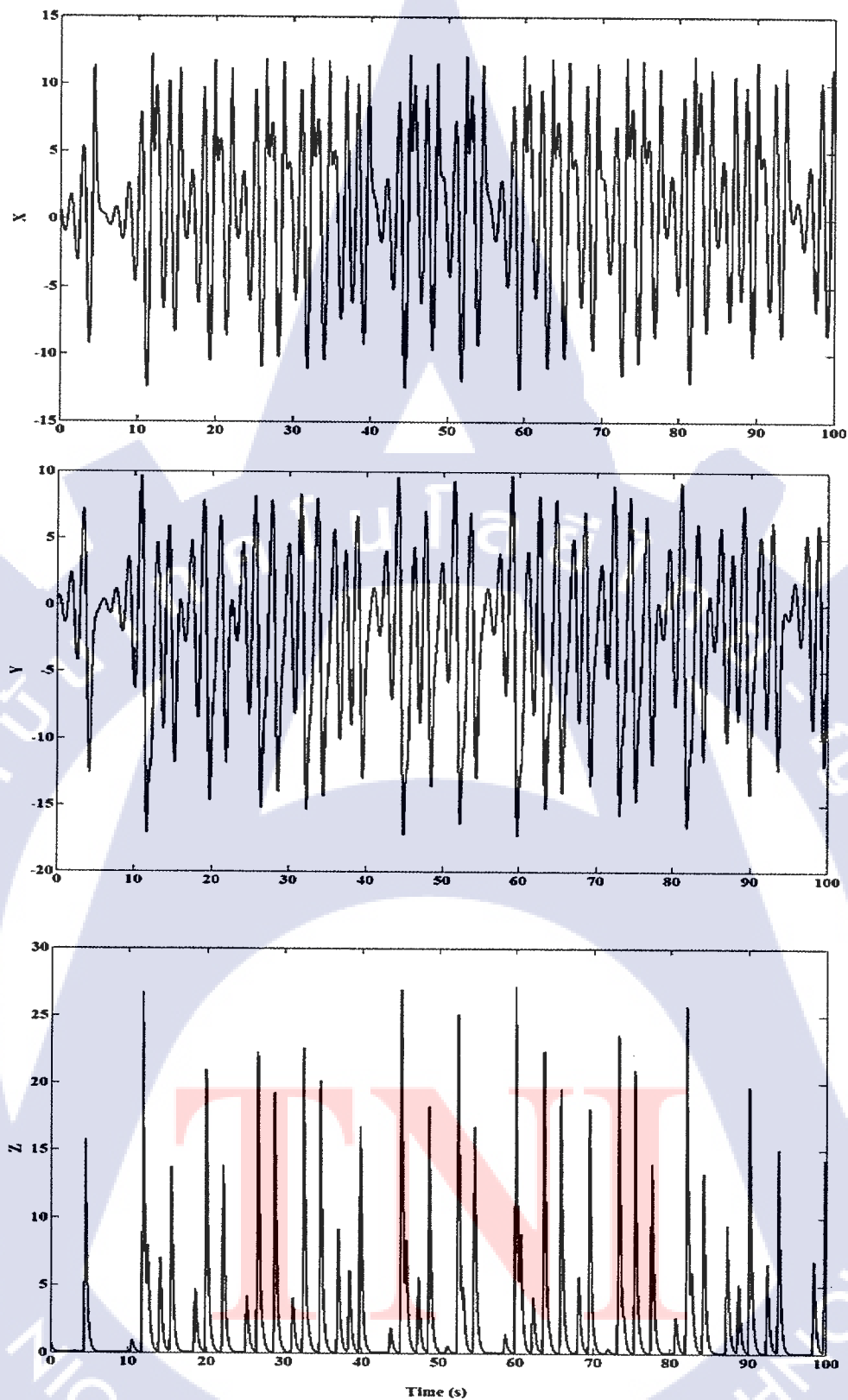


Figure 21 Simulation of time-domain chaotic waveforms of  $x$ ,  $y$  and  $z$ .



## 6. Proposed Secure Communication Systems based on Rössler Attractor

Due to the fact that output signal can recover input signal, it indicates that it is possible to implement secure communication for a chaotic system. Fig. 22 shows the principle scheme of a general secure communication system that employs the masking technique. Figure 23 shows Simulink modeling of chaotic masking communication circuit of the Rössler Attractor. The presence of the chaotic signal between the transmitter and receiver has proposed the use of chaos in secure communication systems. The design of these systems depends on the self-synchronization property of the Rössler Attractor.

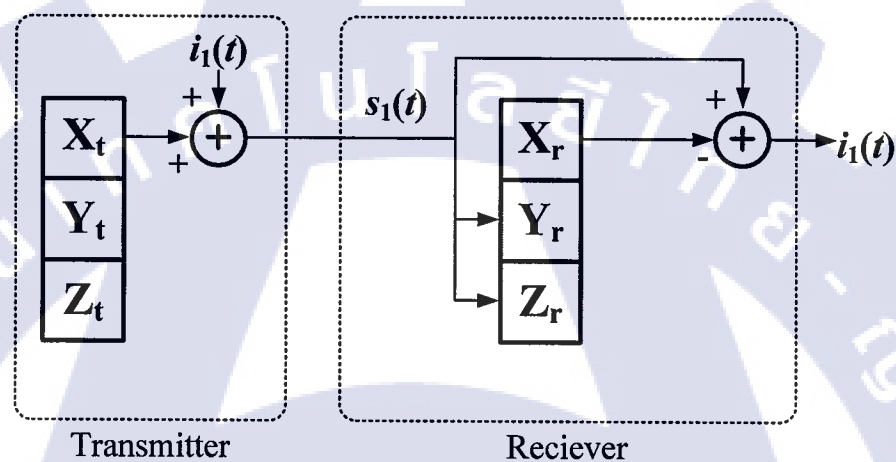


Figure 22 shows the principle scheme of a general secure communication system that employs the masking technique.

Transmitter and receiver systems are identical except for their initial values, in which the transmitter system is 1, 0, 1 and the receiver systems are 3, 0, and 3. It is necessary to make sure the parameters of transmitter and receiver are identical for implementing the chaotic masking communication. In this masking scheme, message signal is added to the synchronizing driving chaotic signal in order to regenerate a clean driving signal at the receiver. Thus, the message has been perfectly recovered by using the signal masking approach through cascading synchronization in the make sure the parameters of transmitter and receivers are identical for implementing the chaotic masking communication. In this masking scheme, a message signal is added to the synchronizing driving chaotic signal in order to regenerate a clean driving signal at the receiver. Thus, the message has been perfectly recovered by using the signal masking approach through cascading synchronization in the Rössler Attractor. Computer simulation results have shown that the performance of Rössler Attractor in chaotic

masking and message recovery. One disadvantage of using one-way coupling method is that compared to this cascading method, it takes longer to synchronize the coupled systems, especially when the coupling parameter is small. This may cause problems in practical applications such as secure communications since information may be delayed or lost during the first period of matching time Rössler Attractor. Computer simulation results have shown that the performance of Rössler Attractor in chaotic masking and message recovery. One disadvantage of using one-way coupling method is that compared to this cascading method, it takes longer to synchronize the coupled systems, especially when the coupling parameter is small. This may cause problems in practical applications such as secure communications since information may be delayed or lost during the first period of matching time.

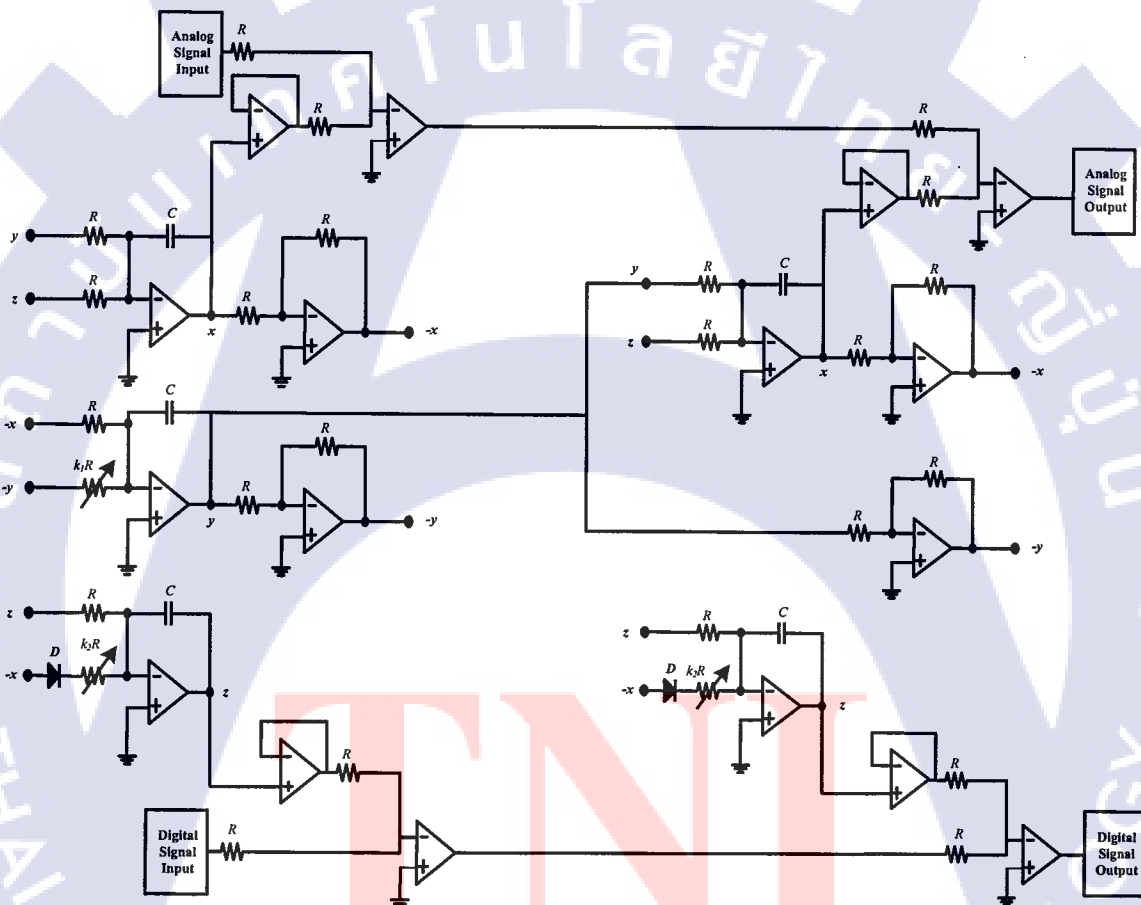


Figure 23 Chaotic Synchronization Circuit.

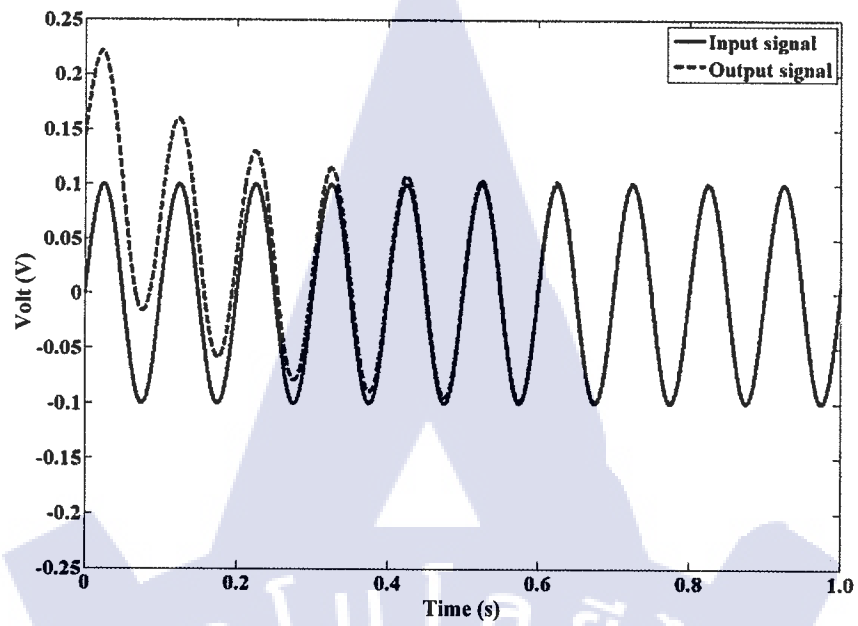


Figure 24 Simulation of Synchronization results input and recovered output signal.

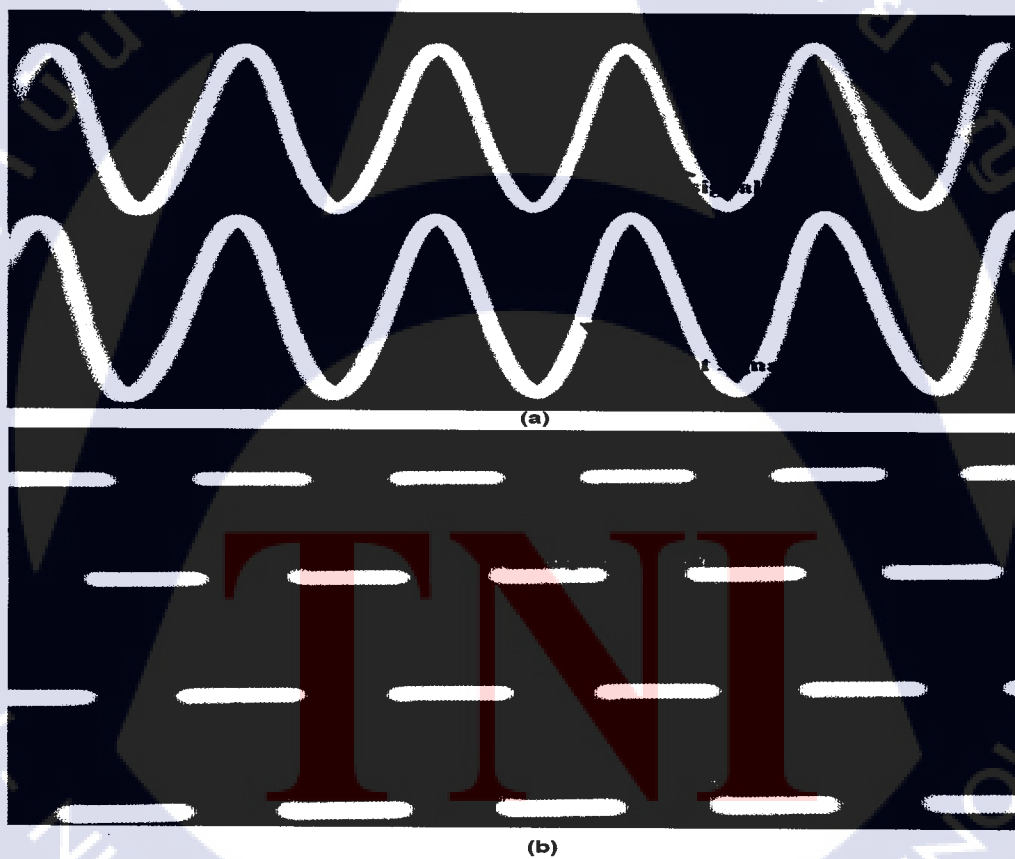
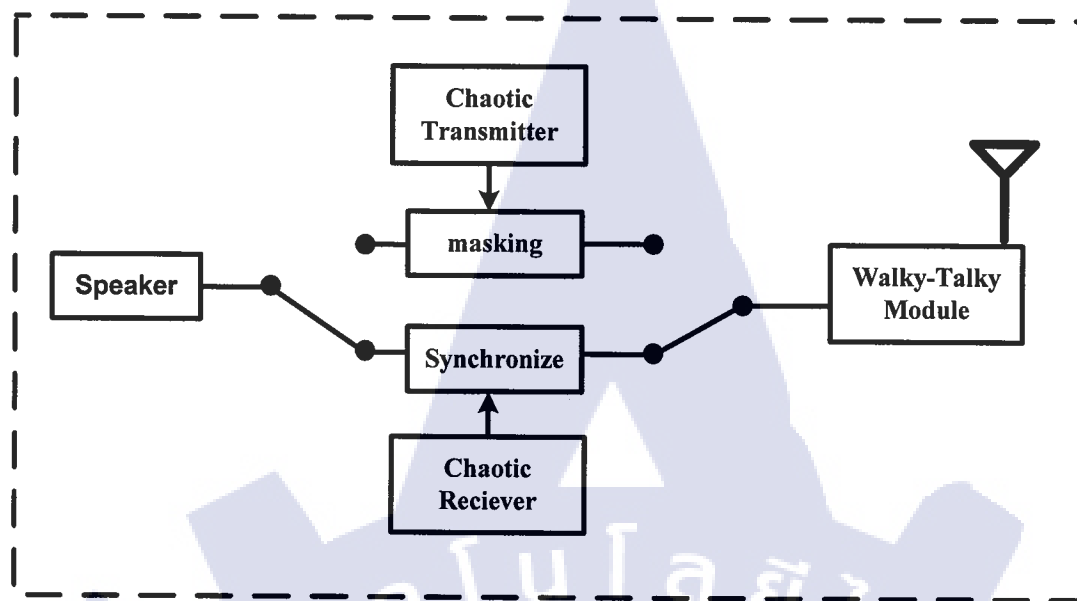
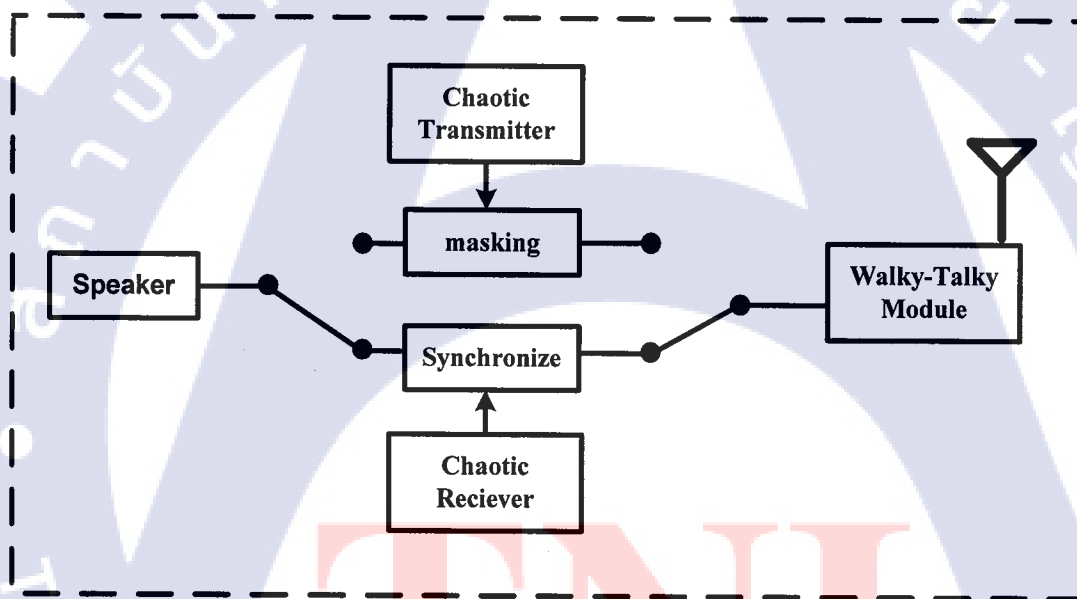


Figure 25 Result chaotic synchronize (a) analog signal and (b) digital signal.



Walky-talky System 1



Walky-talky System 1

Figure 26 Design Schematic of chaotic communication application for walky-talky.

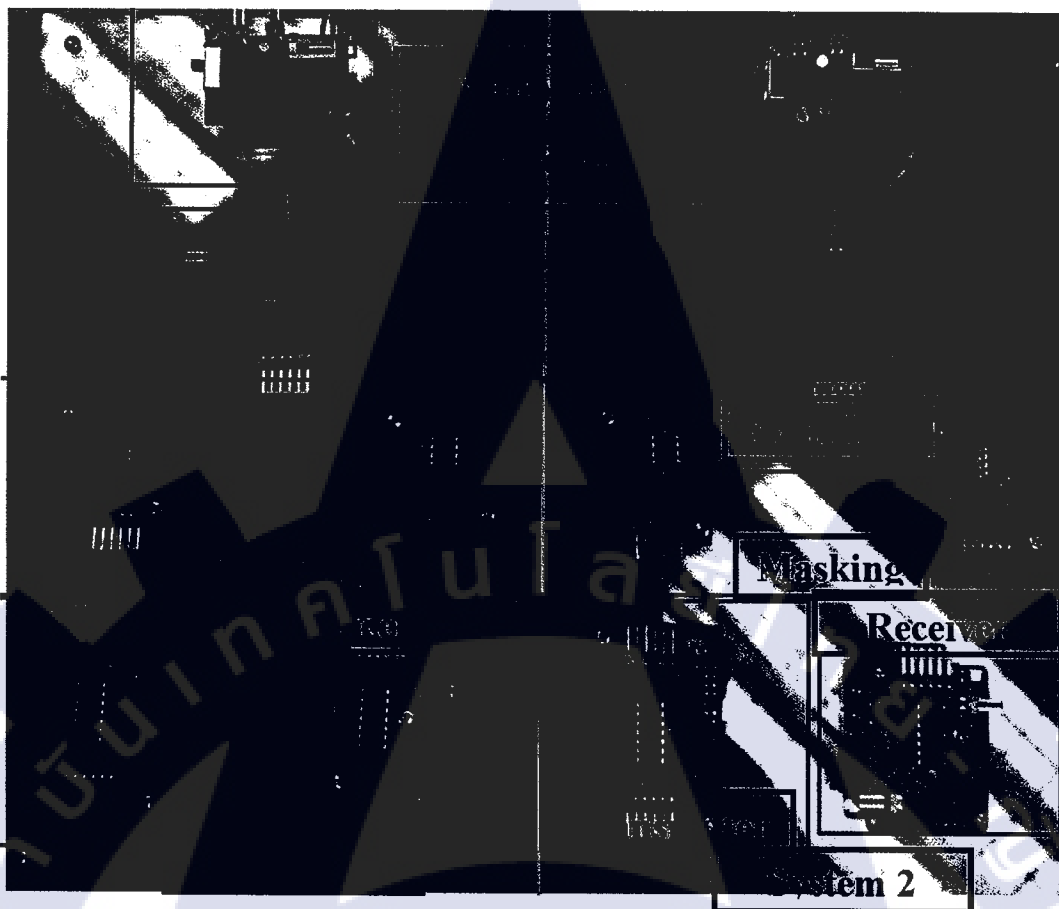


Figure 27 real circuit chaotic communication applications for walky-talky.

## 6. Conclusions

This research focuses on the new Rössler chaotic Attractor's chaotic oscillator circuits and can be described by Bifurcation, Lyapunov Exponents, and Kaplan-Yorke Dimension, implement chaotic circuit and their applications in signal masking communications. New Rössler Attractor's chaotic oscillator circuits has were designed and simulated. Chaotic signal masking circuits were realized using Matlab-Simulink and real circuit. Related figures point out that Matlab-Simulink and real circuit outputs prove the same conclusions. We have demonstrated in simulations that Chaos can be synchronized and applied to secure communications. We suggest that this phenomenon of chaos synchronism may serve as the basis for little



known new Rössler Attractor to achieve secure communication. Simulation results are used to visualize and illustrate the effectiveness of new Rössler chaotic system in signal masking. All simulations results performed on Rössler chaotic system are verified the applicability of secure communication. For experiment Result, we can implement chaotic circuit has high stability and application for two channel chaotic communication. The synchronization of chaotic systems offers an interesting possibility to send secure information via chaotic signals.

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# สถาบันเทคโนโลยีไทย-ญี่ปุ่น Thai-Nichi Institute of Technology 泰日工業大学

สร้างนักคิด ผลิตนักปฏิบัติ สร้างนักประดิษฐ์ ผลิตนักบริหาร

## วารสารสถาบันเทคโนโลยีไทย-ญี่ปุ่น : วิศวกรรมศาสตร์และเทคโนโลยี Journal of Engineering and Technology

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# The Design and Implement of Chaotic Jerk Oscillator with Application to Secure Communication Systems Based on Chaotic Masking

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**Abstract**— Chaotic signals have recently been of much interest as a new promising carrier frequency or an unpredictable masking signal for applications in secure wireless communication systems. Existing chaotic circuits have extensively been constructed based on three-dimensional ordinary differential equations. It has recently been reported that jerk chaotic systems can generate chaotic signals through a single dynamical equation, leading to an algebraically simple mathematical model as well as a cost-effective circuit implementation. This paper presents a very simple autonomous RC chaotic jerk oscillator with nine electronic components. The nonlinearity required for chaos is implemented through the use of a well-known diode equation. Basic dynamical properties are described including equilibria, eigenvalues of Jacobian matrix, chaotic attractors, time-domain waveforms, power spectrum, and bifurcations. Potential application of such a simple autonomous RC chaotic jerk oscillator is presented in message-masking and synchronization for secure digital communications. The results show that the chaotically masked message is fully synchronized at the receiver through the use of very simple circuit. Consequently, the proposed new paradigm on secure communication schemes offers not only a simple mathematical system, but also very cost-effective circuit and system implementations.

**Keywords**—component; Chaotic Jerk Oscillator, Secure Communications, Chaotic Masking

## I. INTRODUCTION

In 1963, Edward Lorenz encountered sensitive dependence of initial conditions of an atmospheric convection model, leading to the discovery of the Lorenz system with seven terms in three-dimensional ordinary differential equations (ODEs) and two quadratic nonlinearities. In 1976, Rössler proposed another chaotic system with seven terms in ODEs and a single quadratic nonlinearity, which is algebraically simpler than the Lorenz system. A single folded-band attractor of Rössler system is also topologically simpler than a twoscroll Lorenz attractor. The Rössler system has therefore been utilized as a basic system in searching for new chaotic systems and employed in various applications such as in secure communications or in control systems. The well-known Chua's circuit [1, 2] is a chaotic oscillator based on three firstorder ordinary differential equations (ODEs). Several chaotic oscillators such as [2], [3], [4], [5], [6], [7] are alternatively based on a single third-order ODE of a jerk form. The term 'jerk' comes from the fact that successive time derivatives of displacement are

velocity, acceleration, and jerk. Most chaotic jerk oscillators have employed nonlinearity only in the  $x$  term of the jerk function. The simplest dissipative chaotic flow has, however, employed a quadratic nonlinearity in the term of the jerk function. Recently, [5] have shown minimal jerk flow of (1) with nonlinearity in the  $\dot{x}$  term of the form

$$\ddot{x} + \dot{x} + x = -\alpha e^{\dot{x}} \quad (1)$$

in which chaos occurs for  $\alpha = 0.27$ . Such a nonlinear function is of particular interest as it resembles diode characteristics. Sprott [6] has subsequently implemented (1) in the form

$$\ddot{x} + \dot{x} + x = -10^{-9} \left\{ e^{\frac{\dot{x}}{0.026}} - 1 \right\} \quad (2)$$

The oscillator (2), however, requires large counts of 14 electronic components including 4 op-amps. Although the oscillator (1) has been implemented by a minimal chaotic jerk equation of (1), the required number of electronic components does not seem to be minimal. It is natural to wonder in the opposite direction whether or not a slightly more complicated chaotic jerk equation may greatly reduce the large counts of electronic components for the chaotic oscillator.

This paper presents a very simple autonomous RC chaotic jerk oscillator with nine electronic components. The nonlinearity required for chaos is implemented through the use of a well-known diode equation. Basic dynamical properties are described including equilibria, eigenvalues of Jacobian matrix, chaotic attractors, time-domain waveforms, power spectrum, and bifurcations. Potential application of such a simple autonomous RC chaotic jerk oscillator is presented in message-masking for secure Communications.

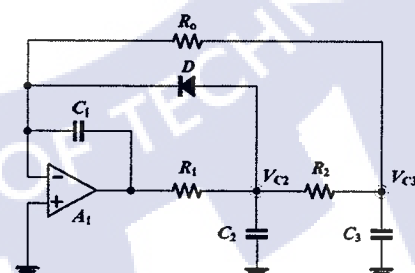


Figure 1. RC-Base chaotic oscillator



The results show that the chaotically masked message is fully synchronized at the receiver through the use of very simple circuit. Consequently, the proposed new paradigm on secure communication schemes offers not only a simple mathematical system, but also very cost-effective circuit and system implementations.

#### Circuit Realizations

In an attempt to reduce the number of electronic components, the minimal form (1) may be modified with a slightly more complicated jerk function and a simple diode equation expressed as

$$I_D = I_S \left\{ \exp \left( \frac{V_D}{nV_T} \right) - 1 \right\} \quad (3)$$

in the following form

$$\ddot{x} = J(\ddot{x}, \dot{x}, x) + I_D R \quad (4)$$

where the voltage drop across the diode is  $V_D = (\dot{x} + 2x)$ ,  $I_S$  is the reverse saturation current of diode,  $V_T$  is the thermal voltage at room temperature,  $n$  is the non-ideality factor of diode,  $R$  is a parameter and  $J(\ddot{x}, \dot{x}, x)$  is a jerk function. Fig.1 illustrates an electronic circuit realization of the autonomous RC chaotic jerk oscillator, consisting of an amplifier, three resistors, three capacitors and a single diode. These components implement an integrator and a second-order RC passive filter in a feedback loop with a smaller nonlinear feedback loop containing a diode. Applying nodal analysis,

$$\begin{bmatrix} \dot{V}_{C3} \\ \dot{V}_{C2} \\ \dot{V}_{C1} \end{bmatrix} = \begin{bmatrix} -2/\tau_0 & 1/\tau_0 & 0 \\ 1/\tau_0 & -2/\tau_0 & 1/\tau_0 \\ -1/\tau_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{C3} \\ V_{C2} \\ V_{C1} \end{bmatrix} + \begin{bmatrix} 0 \\ -I_D/C_2 \\ -I_D/C_1 \end{bmatrix} \quad (5)$$

Equation (5) can be expressed in terms of a normalized dynamical representation through the use of dimensionless variables and parameters as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ -1/A & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ -BI_D \\ -CI_D \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} \dot{x} & x & A \\ \dot{y} & y & B \\ \dot{z} & z & C \end{bmatrix} = \begin{bmatrix} dX/d\tau & V_{C3} & \tau_0/\tau_1 \\ dY/d\tau & V_{C2} & (\tau_0/\tau_2)/nV_T \\ dZ/d\tau & V_{C1} & (\tau_0/\tau_1)/nV_T \end{bmatrix} \quad (7)$$

where the time constants  $\tau_0 = C_2 R = C_3 R$ ,  $\tau_1 = C_1 R$  and  $\tau = t/\tau_0$ . It is seen from (7) that  $X = (\dot{X} + 2X)$  and consequently the diode equation  $I_D$  can be expressed as  $I_D = I_S \{ \exp[V_{C2}/(nV_T)] - 1 \} = I_S \{ \exp[Y] - 1 \}$ , resulting in

$$I_S \{ \exp[\dot{X} + 2X] - 1 \} = i_D - I_S \quad (8)$$

Where  $I_S \exp(Y) = I_S \exp(\dot{X} + 2X)$ . Alternatively, the dynamical representation in (6) can also be written in a jerk representation as

$$\ddot{x} = -\ddot{X} (4 + BI_D) - \dot{X} (3 + 2BI_D) + AX - CI_D \quad (9)$$

It is obvious that a jerk model in (11) is described in the form shown in (4) as  $\ddot{x} = J(\ddot{x}, \dot{x}, x) + I_D R$ . In addition, the proposed circuit shown in Fig.1 has been designed with components  $R = 1 \text{ k}\Omega$ ,  $C_1 = 0.1 \text{ }\mu\text{F}$ ,  $C_2 = C_3 = 10 \text{ }\mu\text{F}$ ,  $R_0$  is a potentiometer. The Diode model is 1N4148 where  $I_S = 14.11 \times 10^{-9}$ ,  $n = 1.984$ , and  $V_T = 25.85 \times 10^{-3}$ . The counting number of electronic components is only 8 and is therefore reduced by 43% compared to that of 14 counts in [6]. In particular, only a single op-amp is necessary for circuit implementation.

## II. DYNAMICAL PROPERTY ANALYSIS AND NUMERICAL SIMULATIONS

Dynamic properties were mathematically analyzed using nonlinear theorems and numerically investigated using MATLAB. The initial condition was set at (0.1, 0, 0), which lies in the basic of attractor. The chaotic behaviors were simulated using the Fourth-order Runge-Kutta method with time step size of  $5 \times 10^{-6}$ . The system (7) is invariance under the transform  $(x, y, z) \rightarrow (-x, -y, z)$ , i.e. is symmetric around the  $z$ -axis and remains confined to the positive half-space with respect to the  $z$  state variable. The divergence of flow of the dynamic system is described as

$$\nabla \cdot V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -\frac{4}{\tau_0} = -4 \times 10^3 \quad (10)$$

Therefore, the chaotic system (7) is a dissipative system with an exponential rate of contraction as

$$\frac{dv}{dt} = \exp \left( -\frac{4}{\tau_0} \right) = \exp(-4 \times 10^3) \quad (11)$$

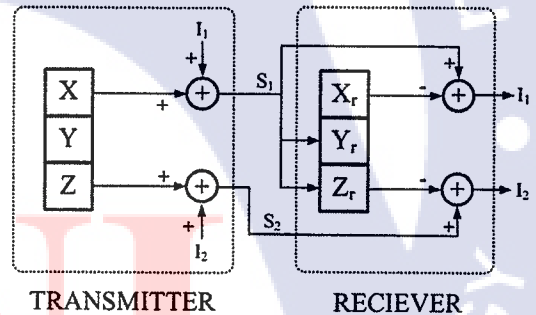


Figure2. Principle scheme of a general secure communication system with masking technique

In other words, a volume element  $V_0$  becomes smaller by the flow in time  $t$  into a volume element  $V_0 \exp(-t)$ . Each volume containing the trajectories shrinks to zero as  $t \rightarrow \infty$  at an exponential rate of  $-4 \times 10^3$ . System orbits are ultimately confined into a specific limit set of zero volume, and the system asymptotic motion settles onto an attractor of the system. In order to investigate the linear stability, the system (5) was linearized, and a single fixed point was found at (0, 0, 0). The Jacobian matrix of partial derivatives is defined as

$$J = \begin{bmatrix} -2/\tau_0 & 1/\tau_0 & 0 \\ 1/\tau_0 & -(2/\tau_0) - \beta_{C2} & 1/\tau_0 \\ -1/\tau_1 & -\beta_{C1} & 0 \end{bmatrix} \quad (12)$$

where the parameters  $\beta_{C1} = \{I_s \exp(V_{C2}/nV_T)\}/nV_T C_1$  and  $\beta_{C2} = \{I_s \exp(V_{C2}/nV_T)\}/nV_T C_2$ . The resulting eigenvalues of the Jacobian matrix in (14) evaluated at the fixed point are consequently equal to

$$\begin{aligned}\lambda_1 &= -6.1531 \\ \lambda_2 &= 1.0765 + 3.8852i \\ \lambda_3 &= 1.0765 - 3.8852i\end{aligned}\quad (13)$$

It is evident from (13) that the fixed point is a saddle focus node as the eigenvalue  $\lambda_1$  is negative real value and  $\lambda_{2,3}$  are a pair of complex conjugate eigenvalues with positive real parts.

### III. SECURE COMMUNICATION SYSTEMS BASE ON CHAOTIC MASKING

There are number of possible methods that have been developed for synchronization in chaotic communications. In the masking method, synchronization is achieved by simply if the conditional Lyapunov exponents for the systems are negative for the given operating parameters. Thus, one could simply recover the message signal from the received chaotic signal through by means of a subtraction at the receiver. This synchronization is robust against small perturbations of the carrier signal. In the chaotic modulation method the message signal becomes part of the dynamics, which is more robust because of the greater symmetry between chaotic oscillator and response. In the chaos shift keying technique the message information is encoded onto the attractor by means of modulating a parameter of the chaotic oscillator, typically in a

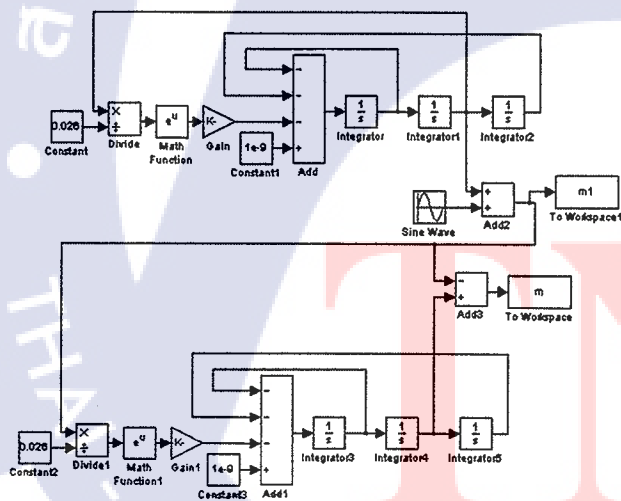


Figure 3. Bifurcation diagram of the output  $V_{C3}(x)$  versus the bifurcating resistor  $R_0$ .

binary manner. In all these three schemes synchronization is an obvious way of recovering the original information. Fig.2 shows the principle scheme of a general secure communication system with masking technique the transmitter can be used as a single drive system for a dual-channel transmitter independent of its response subsystem at the receiver.

### IV. SIMULATIONS AND EXPERIMENTAL RESULTS

Fig.3 shows the simulation schematics in MATLAB. Electronic circuit simulation was performed in PSpice while the experiment was conducted on board using discrete components. The components value were set at  $R = 1 \text{ k}\Omega$ ,  $C_1 = 0.1 \text{ }\mu\text{F}$ ,  $C_2 = C_3 = 10 \text{ }\mu\text{F}$ , Diode is 1N4148 and an Op-amp is LM741,  $R_0$  is  $1.2 \text{ k}\Omega$ . Fig. 5 shows the consistent chaotic attractors in x-y plane obtained from PSpice simulation and experiment. It is obvious that the proposed chaotic system circuit truly possesses chaotic behaviors with a single folded-band topology orbiting around the fixed point at (0, 0, 0). Fig.4 shows the bifurcation diagram of the peak of  $V_{C3}$  (X) versus the parameter  $R_0$ , exhibiting a route to chaos. I

t is seen that the bifurcating parameter  $R_0$  can be tuned for chaos in wide region of  $0.8 \text{ k}\Omega$  to  $3 \text{ k}\Omega$ . Upon setting  $R_0$  to a value of  $1.2 \text{ k}\Omega$ , the chaotic attractors are displayed in Fig. 3 for a three-dimensional view, an x-y phase plane, an x-z phase plane, and a y-z phase plane. The attractor of three-dimensional view remains confined to the positive half-space of the z-axis. Fig.6 shows the experiment chaotic attractors in x-z plane. Fig.7 shows the synchronization results, (a) input and recovered output signal at the receiver, (b) synchronization errors. It is seen from Fig. 7 that the masked signal can be retrieved shortly with low errors. In addition, other types of signals such as rectangular or common human speech can also be applied to this method.

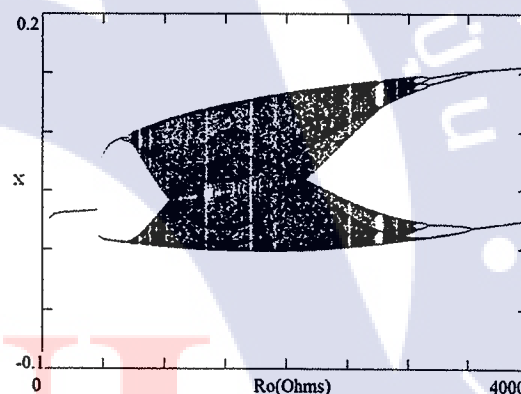


Figure 4. Bifurcation diagram of the output  $V_{C3}(X)$  versus the bifurcating resistor  $R_0$ .

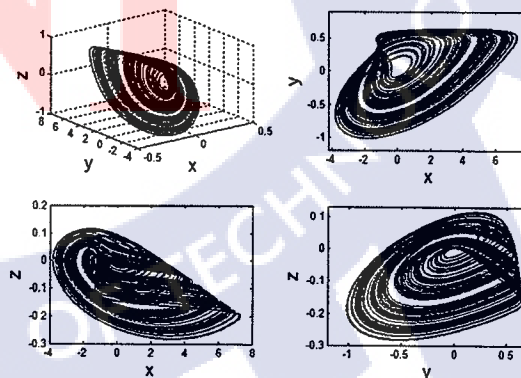


Figure 5. Chaotic attractors in three-dimensional view, an x-y plane, an x-z plane, and a y-z plane.





Figure 6. Experiment chaotic attractors in  $x$ - $z$  plane

## V. CONCLUSIONS

This paper has presented a very simple autonomous RC chaotic jerk oscillator with nine electronic components. The nonlinearity required for chaos is implemented through the use of a well-known diode equation. Basic dynamical properties are described including equilibria, eigenvalues of Jacobian matrix, chaotic attractors, time-domain waveforms and bifurcations. Potential application of such a simple

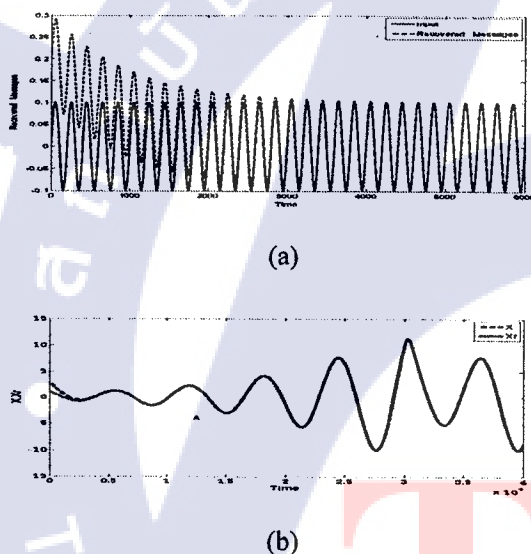


Figure 7. Synchronization results; (a) input and recovered output signal at the receiver, (b) synchronization errors.

autonomous RC chaotic jerk oscillator is presented in message-masking and synchronization for secure digital communications. The results show that the chaotically masked message is fully synchronized at the receiver through the use of very simple circuit. Consequently, the proposed new paradigm on secure communication schemes offers not only a simple mathematical system, but also very cost-effective circuit and system implementations.

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# Implementation of Rössler Chaotic System through Inherent Exponential Nonlinearity of a Diode with Two-Channel Chaotic Synchronization Applications

S. Larptwee and W. San-Um

**Abstract**—This paper presents a new Rössler chaotic system using exponential nonlinearity and its application to two-channel synchronization. The proposed chaotic system exhibits a chaotic attractor that resembles the original Rössler system with only six-term in three-dimensional ordinary equation systems using the exponential nonlinearity. Chaotic dynamics are described in terms of equilibria, Jacobian matrix, time domain waveforms, chaotic attractors, and bifurcation diagram. The circuit implementation is relatively compact and simple since the exponential nonlinearity can be achieved by an inherent nonlinearity of single diode. An application to a two-channel secure communication are also demonstrated, showing a fast, low-error and robust synchronization processes.

## I. INTRODUCTION

EDWARD LORENZ[1] encountered sensitively dependent initial conditions of an atmospheric convection model while performing numerical simulations in 1963 leading to the discovery of the Lorenz system with seven-terms in three-dimensional ordinary differential equations and two quadratic nonlinearities. In 1976, Rössler [2] proposed a chaotic system with seven terms and a single quadratic nonlinearity, which is algebraically simpler than Lorenz system. In addition, a single folded-band attractor of Rössler system is topologically simpler than a two-scroll Lorenz attractor. Such Lorenz and Rössler systems have consequently led to considerable research interests in searching for new chaotic systems with fewer terms in ODEs or more complex attractor topology.

Several chaotic systems with fewer than seven terms and two quadratic nonlinearities continuously been reported as variants in Lorenz system family [3-7]. Complex three-scroll and four-scroll attractors based on Lorenz system have also been suggested through the use of three or more quadratic nonlinearities [8-10]. On the other hand, simple chaotic systems with a single nonlinearity similar to Rössler system are rarely found. In fact, Rössler himself had proposed another system with six terms and a single quadratic nonlinearity in 1979. In 1994, Sprott [4] found fourteen cases with six terms and a single quadratic nonlinearity through an intensive numerical computer search. Recently, many simple systems have been proposed in simple Jerk

equations with single quadratic or non-quadratic nonlinearities. Despite the fact that these simple Jerk chaotic systems with a single nonlinearity potentially resemble the single folded-band Rössler attractor, the Kapelan-York [4] dimension ( $D_{KY}$ ) as a measure of complexity is somewhat lower than the original Rössler attractor that possesses the greatest value of  $D_{KY} = 2.1587$ . This leads to a question of whether the original Rössler system in dynamic forms can be simplified into fewer terms with simple nonlinearity, or modified for more complex attractor [12-15]. No simplifications of Rössler system has never been found so far. This paper therefore presents a new Rössler chaotic system using exponential nonlinearity and its application to two-channel synchronization. The proposed chaotic system exhibits a chaotic attractor that resembles the original Rössler system with only six-term in three-dimensional ordinary equation systems using the exponential nonlinearity. An application to a two-channel secure communication are also demonstrated, showing a fast, low-error and robust synchronization processes.

## II. PROPOSED NEW RÖSSLER CHAOTIC SYSTEM USING EXPONENTIAL NONLINEARITY

Based on the Rössler system proposed in 1979 [7], the first and the second equations, i.e.  $\dot{x} = -y - z$  and  $\dot{y} = x + ay$ , initiate a normal band of the attractor through an outward spiral motion into the  $x$ - $y$  phase plane. Nonlinear interactions between  $x$  and  $z$  variables in the third equation, i.e.  $\dot{z} = b + z(x - c)$ , form an additional folded band to the overall attractor. It is noticeable that the folded band in Rössler attractor rises and returns exponentially in  $z$ -dimension especially for positive values of  $x$  variable under the flows. This aspect implies that the third equation may be modified through the use of an exponential nonlinearity. Therefore, a new chaotic system is therefore presented in three-dimensional autonomous ODEs expressed as

$$\begin{aligned}\dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= -z + bF(x)\end{aligned}\tag{1}$$

where  $(x, y, z) \in \mathbb{R}^3$  are dynamical variables,  $(a, b) \in \mathbb{R}^+$  are system parameters, and  $F(x)$  is a nonlinear function required for chaos. A particularly simple case of the nonlinear function  $F(x)$  is presented using exponential functions. In other words,

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$$F(x) = \exp(x) \quad (2)$$

It can be considered from (2) such an exponential nonlinearity can be implemented through the use of a diode instead of using a complicated voltage multiplier circuit such as AD633 chip.

### III. DYNAMICS ANALYSIS

The system (1) maintains the first and the second equations of Rössler system while the third equation has been simplified into two distinct terms, including the linear term  $-z$  and the nonlinear term  $bF(x)$ . The existence of attractor can be described by the divergence of flows as

$$\nabla \cdot V = \partial \dot{x} / \partial x + \partial \dot{y} / \partial y + \partial \dot{z} / \partial z = a - 1. \quad (3)$$

For a dissipative chaotic system,  $p < 0$  and therefore  $a$  is limited into the region  $0 < a < 1$ . The exponential rate of contraction is  $dV/dt = \exp(a-1)$  and hence a volume element  $V_0$  is contracted in time  $t$  by the flows into a volume element  $V_0 \exp(-t)$ . Each volume containing the system trajectories shrinks to zero as time  $t$  approaches  $+\infty$ . All system orbits will be confined to a specific limit set of zero volume, and the asymptotic motion converges onto an attractor. It can be concluded that the existence of attractors is constant and independent to the nonlinear term  $bF(x)$ . For the equilibrium analysis, linearizing (1) by setting the system of equation equals zero, i.e.

$$\begin{aligned} 0 &= -y - z \\ 0 &= x + ay \\ 0 &= -z + be^x \end{aligned} \quad (4)$$

The system (4) has a single equilibrium point at  $(0, 0, 0)$  and the Jacobian of the system is

$$J = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ bF' & 0 & -1 \end{bmatrix} \quad (5)$$

Applying the equilibrium point P into this Jacobian matrix and analyzing  $|J\lambda - J| = 0$  reveals a resulting characteristic polynomial as follows:

$$\lambda^3 + (1-a)\lambda^2 + (bF' - a + 1)\lambda + (abF' - 1) = 0 \quad (6)$$

According to the Routh–Hurwitz [9] stability criterion, the system (1) is unstable when  $F' < (1 + (1-a)2)/(b-2ab)$ . Note that dynamic behaviors depend on two parameters  $a$  and  $b$ , and can be characterized completely by the plot of parameter space without redundancy. For all particular values of  $a$  and  $b$  in the subsequent numerical analyses, the resulting eigenvalues  $\lambda_1$  is a positive real number and  $\lambda_2$  and  $\lambda_3$  are a pair of complex conjugate with positive real parts, indicating that the equilibrium points are all saddle focus points.

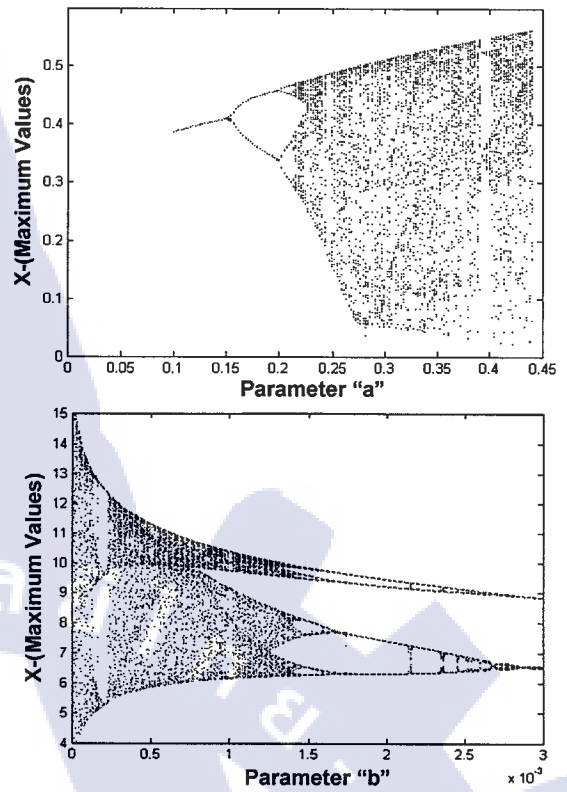


Fig.1. Bifurcation diagrams exhibiting a route to chaos.

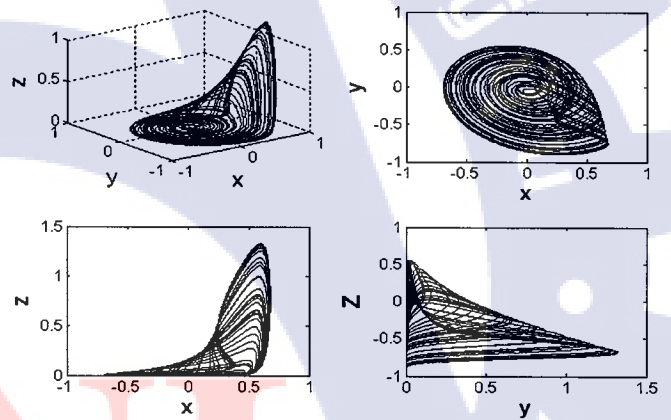


Fig.2. Simulation Phase portraits with  $a=0.30$  and  $b=0.0007$ ,  $LEs = (0.0638, 0, -0.8641)$ ,  $D_{KY} = 2.0738$ .

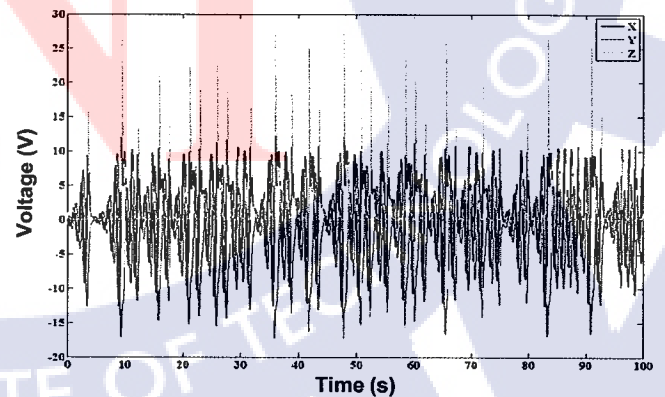


Fig.3. Time-domain waveforms showing chaotic behaviors.

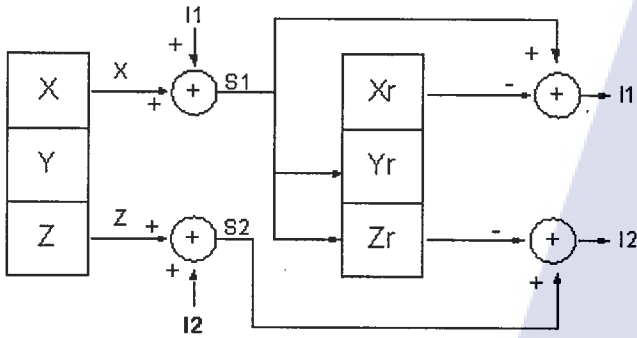


Fig. 4. The block diagrams of the transmitter and receiver of two-channel chaotic synchronizations.

Numerical simulations have been performed in MATLAB using the initial condition of  $(x_0, y_0, z_0) = (1, 0, 1)$ . In fact, the initial condition is not crucial, and can be selected from any point that lies in the basin of attractor. In order to find the control parameter that offers the maximum values of chaoticity and complexity, Fig. 1 shows the bifurcation diagram of the peak of  $z$  ( $z_{\max}$ ) versus the parameters  $a$  and  $b$ . It is seen in Fig. 1 that the system exhibits a period-doubling route to chaos. As for particular illustrations, the control parameter at  $a=0.35$  and  $b=0.0007$  is chosen in simulations of dynamical behaviors. The chaotic attractors are displayed in Figs. 2(a), 2(b), 2(c) and 2(d) for a three-dimensional view, an  $x$ - $y$  phase plane, an  $x$ - $z$  phase plane and a  $y$ - $z$  phase plane, respectively. It is apparent in Fig. 2 that the attractor of the proposed system has a single-scroll topology, and potentially resembles the existing Rössler attractors in all phase planes. Fig. 3 shows apparently chaotic waveforms in time domain. It can be seen that the three signals are random in both amplitudes and frequencies. The DC offset is zero since the equilibrium point is at  $(0, 0, 0)$ .

#### IV. APPLICATIONS TO TWO-CHANNEL SECURE COMMUNICATION SYSTEMS

The chaotic synchronizations provide two input and two output messages [16-23]. Fig. 4 shows the block diagrams of the transmitter and receiver of two-channel chaotic synchronizations. At the transmitter, a modified Rössler attractor described in (1) can be used as a single drive system for a dual-channel transmitter independent of its response subsystem at the receiver as follows:

$$\begin{aligned}\dot{x} &= -y - z \\ \dot{y} &= x + 0.35y \\ \dot{z} &= 0.0007e^x - z\end{aligned}\quad (7)$$

Based on such a modified rössler system using diode equation as shown in Fig. 4, the dual channel transmitter consists of two transmitter and receiver signals. The first transmitter signal is  $s_1(t) = x_t(t) + i_1(t)$  where  $x_t(t)$  is a chaotic signal and  $i_1(t)$  represents the first original input which is transmitted. The second transmitter signal is  $s_2(t) = z_t(t) + i_2(t)$  where  $z_t(t)$  is a chaotic masking signal and  $i_2(t)$

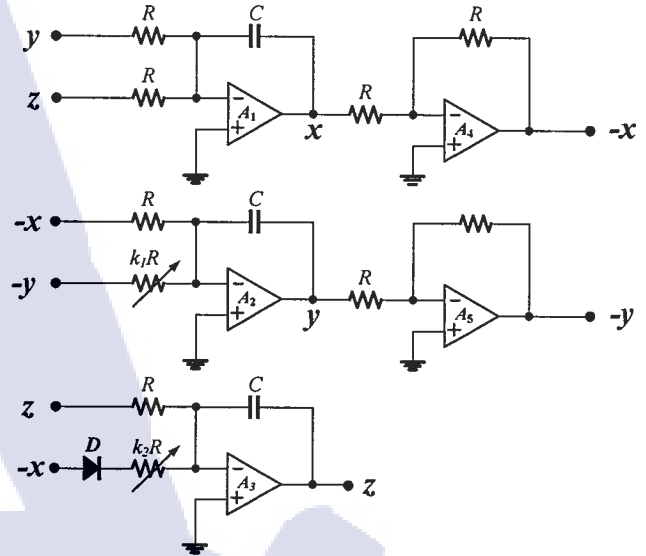


Fig 5. Circuit design of the proposed Rössler chaotic system using exponential nonlinearity.

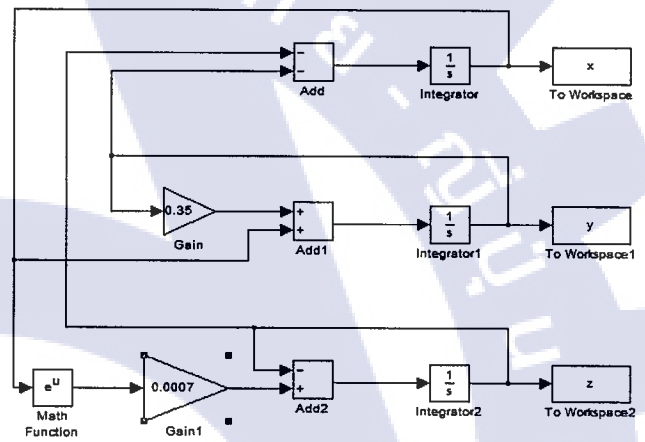


Fig 6. Simulink model of the proposed Rössler chaotic system using exponential nonlinearity.

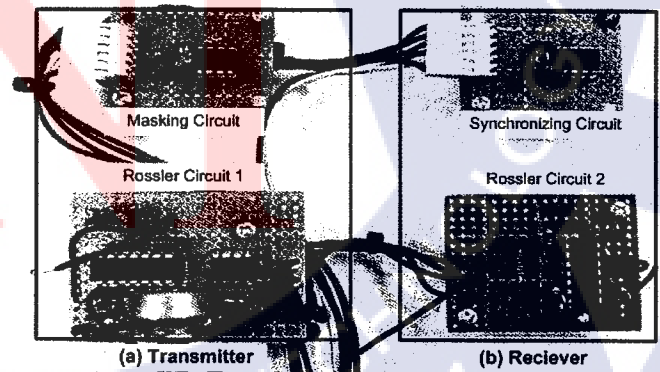


Fig 7. Circuit implementations of the proposed chaotic system and its synchronization systems.

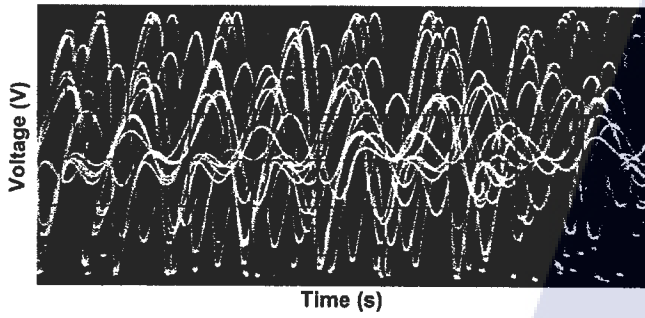


Fig 8. Measured chaotic output signals

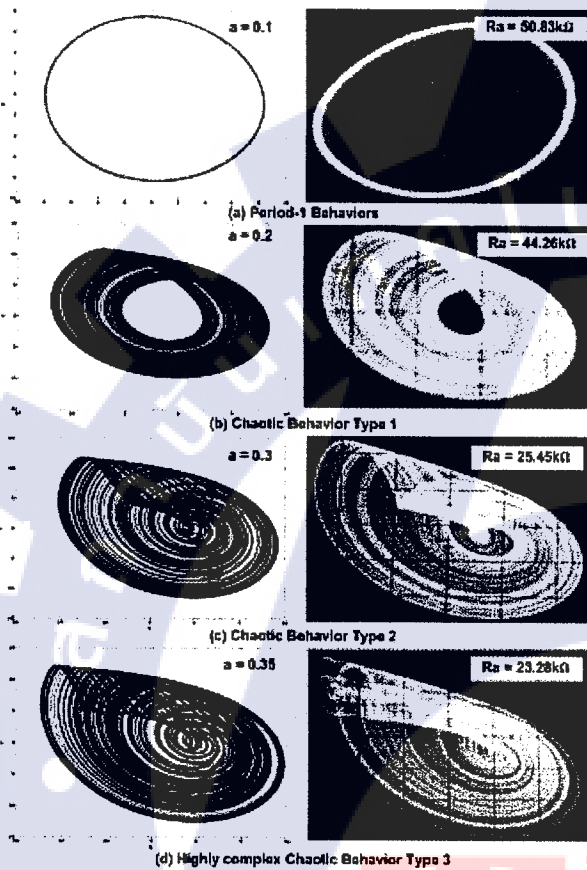


Fig 9. Simulated and measured chaotic attractors at different parameter values.

represents the second original message which is transmitter. At the receiver, a modified rössler attractor described in (1) can be used as single response subsystem for dual-channel receiver as follow:

$$\begin{aligned}\dot{x}_r &= -y_r - z_r \\ \dot{y}_r &= x_r + 0.35y_r \\ \dot{z}_r &= 0.0007e^{x_r} - z_r\end{aligned}\quad (8)$$

With reference to Fig.4, the dual-channel receiver consist of produces a cmasking signal  $x_r(t)$  and  $z_r(t)$ , respectively when the receiver synchronizes with  $s_2(t)$ , then  $x_r(t) \approx x_t(t)$ . The input signal  $i_1(t)$  can be recovered as  $\hat{i}_1(t) = s_1(t) - x_r(t) = x_t(t) + i_1(t) - x_r(t) \approx i_1(t)$ .

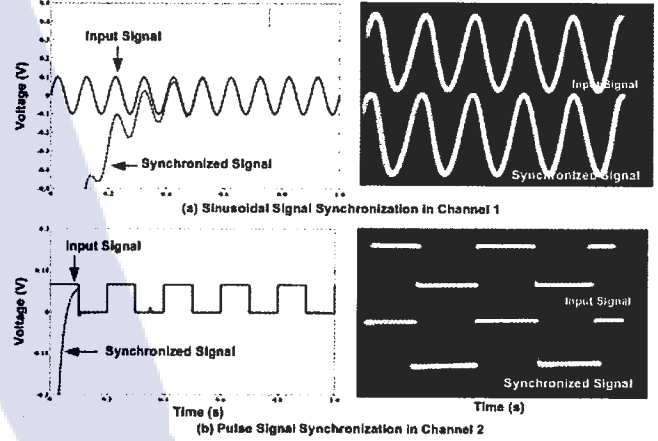


Fig 10. Simulated and measured synchronized signals in two channels.

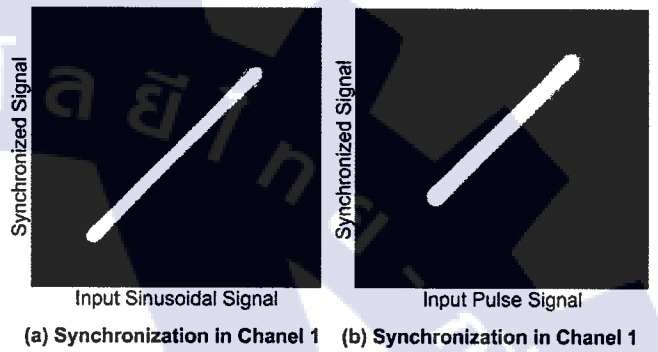


Fig 11. Phase-space plots of two channel signals showing a highly correlated transmitted and received signals.

Similarly when the receiver synchronizes with  $s_1(t)$ , then  $z_r(t) \approx z_t(t)$ . The input signal  $i_2(t)$  can be recovered as  $\hat{i}_2(t) = s_2(t) - z_r(t) = z_t(t) + i_2(t) - z_r(t) \approx i_2(t)$ . As for a simple example, the first transmitter input,  $i_1(t) = 0.1 \sin(2\pi f_1 t)$ , where the frequency  $f_1 = 1000$  Hz and the second transmitted input  $i_2(t)$  is a pulse-train rectangular wave form with an amplitude 0.1 and the frequency  $f_2 = 1000$  Hz. Self-synchronize can be achieved over a wide range of initial condition e.g. at the transmitter  $[x_t(0), y_t(0), z_t(0)] = [1, 0, 1]$  and at the receiver  $[x_r(0), y_r(0), z_r(0)] = [3, 0, 3]$ .

## V. EXPERIMENTAL RESULTS

Fig.5 shows the circuit diagram of the proposed new Rössler chaotic system using exponential nonlinearity. The circuit consists of three integrator section, including op-amps  $A_1, A_2$  and  $A_3$ . The inverting amplifiers are  $A_4$  and  $A_5$ . All operational amplifiers are implemented by TL084CN with 9-V power supply. The diode is IN4001. The state equations from nodal analysis is given by where  $k_1 R = 23.28$  KΩ, and  $k_2 R = 0.7$  KΩ are scaling parameters. The value of the  $R = 10$  KΩ and  $C = 1$  nF. In order to verify in terms of block diagram, Fig 6 shows the Simulink model of the proposed Rössler chaotic system using exponential nonlinearity when  $a = 0.35$  and  $b = 0.0007$ . Fig 7 shows circuit implementations of the proposed chaotic system and its synchronization systems. Fig 8 depicts the measured chaotic output signals. Fig 9 shows the simulated and measured chaotic attractors at different parameter values. It



can be seen from Figs. 8 and 9 that the results from both simulations and experiments are closely resemble. Fig 10 shows the simulated and measured synchronized signals in two channels. Fig 11 shows phase-space plots of two channel signals showing a highly correlated transmitted and received signals, indicating that the two signals are completely synchronized.

## CONCLUSIONS

This paper has presented a new Rössler chaotic system using exponential nonlinearity and its application to two-channel synchronization. In comparisons to other existing implementation of Rössler-based chaotic system, the proposed chaotic system exhibits a chaotic attractor that resembles the original Rössler system with only six-term in three-dimensional ordinary equation systems using the exponential nonlinearity. All dynamical behaviors were investigated through equilibria, Jacobian matrix, time domain waveforms, chaotic attractors, and bifurcation diagram. Cost-effective implementations of chaotic circuits were based on linear op-amps, capacitors, resistors, and a single diode employed as a nonlinear component. An application to a two-channel secure communication was also demonstrated through sinusoidal and pulse signals. The synchronization could recover the transmitted signals with fast and robust synchronization processes. The proposed circuit offers a potential alternative to robust cost-effective nonlinear oscillators in communications and controls applications.

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