The Design and Implementation of Weak Signal Detection Methods using Duffing Chaotic Oscillator

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CHIRAMATHE NAMI : THE DESIGN AND IMPLEMENTATION OF WEAK SIGNAL DETECTION METHOD USING DUFFING CHAOTIC OSCILLATOR, ADVISOR: DR. WIMOL SAN-UM,77PP.

This thesis presents weak signal detection methods using Duffing chaotic oscillator. The dynamic characteristics and properties of Duffing chaotic oscillator under the detection mode are mathematically analyzed, including an existence of attractor, equilibrium points, Jacobian matrices, bifurcations, chaotic waveforms, and frequency-domain spectrums. As the precision of the system depend on the parameter threshold value still unsolved and a small variation of parameter value may cause a dramatically change in the chaotic system behavior, this thesis has contributed two major significant research outcomes, including parameter optimization and hardware implementation of the Duffing chaotic oscillator. The proposed parameter optimizations aim to achieve the parameter robustness for circuit operation under weak signal detection mode through the comparisons of Kaplan-Yorke conjecture that quantitatively measures the system complexity. The optimized parameters of Duffing equation has been found and employed for the circuit implementation. The circuit implementation of Duffing chaotic oscillator has also been proposed with minimal components with band-pass filtered output and current measurement circuit. The circuit could potentially measure weak signals under noisy conditions through the phase-plane observation in the changes of chaotic attractor. The proposed weak signal detection method can be used as an alternative to cost-effective weak signal detection systems in a variety of applications such as in fault location, failure monitoring systems, and communications systems.

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Chapter 1

Introduction

This chapter introduces Weak Signal Detection (WSD) principles and methods. Some related techniques of weak signal detection methods are studied, including Stochastic Resonance (SR), a detection algorithm base on Wigner-Ville Distribution (WVD) and also a circuit implementation CW NMR broad-line spectrometer. Statement of problems, hypothesis, research scope and expected outcomes are also included.

1.1 Weak Signal Detection

Weak Signal Detection (WSD) has been extensively utilized in many signal processing fields such as communications, mechanical industry, medical science and military. In the industry area, fault detection in the incipient stage of failure appearing has been gaining importance in recent time, due to the fact that motors has widely applied in most industrial production. Through monitoring the states of asynchronous motor, weak signal detection can detect incipient failure and prevent the further deterioration of the fault and reduce the economic losses that the sudden fault causes, avoiding the threat to the staffs and equipment. It is the reason why diagnose reliably in the incipient failure and identifying the type of fault has great important practical value. In communication area, WSD system was widely implemented in electronic components including a communications receiver, radar, sonar and other areas etc.

An existing problem encounter within a radio communications is the loss of signal in complex propagation environments such as large buildings, tunnels, basements, and collapsed structures. Signal strength was reduced due to attenuation through building materials can significantly hamper communication.In Medical area, noise always influences on the result of signal-detection. The detecting signal under the low signal-to-noise ratio (SNR) form body signals are widely observed medical-signal detection field. However, it is an intractable problem of signal-detection to extract weak signal under a high noisy environment. Adaptation filter provides a simple and useful way to detect signal through the measurement and detection. The possible method to extract the best combination of weak signal under the condition of weak signal, Wavelet Transform (WT) has been employed to detect the ECG signal. Furthermore, There have a number of methods were used to determine the weak signal detection methods such as (1) Stochastic Resonance (SR)[1], (2) A detection algorithm base on Wigner-Ville Distribution (WVD)[2] and (3) also a circuit implementation CW NMR broad-line spectrometer[3].

First, The Stochastic Resonance (SR) method is a phenomenon which small periodic signals immersed in large background noise can be detected. The most important quantifier for SR is the Signal to Noise Ratio (SNR) which passes through a maximum as a function of the noise amplitude for systems showing SR. Using the bimodal cubic-map, it is shown that SNR can be enhanced by a suitable coupling of two systems capable of SR. The classical SR deals with the detection of a single sub-threshold signal immersed in noise. However, in practical situations a composite signal consisting of two or more harmonic components in the presence of background noise is encountered. For example, signals impinging on sensory neurons often have multiple discrete spectral lines, as in the case of human speech and musical tones. Moreover, two frequency signals are widely used in communications, laser physics and acoustics. As a preliminary model for such situations, the method is important to analyze how a typical bi-stable system responds to similar inputs and noise. The SR method can be used as an effective tool in signal transmission and in communication in noisy environments by using noise as a design parameter and tuning the background noise level. All the basic frequencies contained in a signal can be enhanced and separated out using suitable filters.

Second, the detection algorithm base on Wigner-Ville Distribution is consisting of phase-space reconstruction technique and principal components analysis. The detection method of weak signals can be employed in noisy environments. With the algorithm, the frequency of the signal can be extracted even when the S/N reaches negative value and the FFT power spectrum shows no trace of its spectral characteristics. The signal detection scheme is insensitive to the nature of the background noise. The basic idea of the detection method is to transform, or map an input time series into a 2-dimensional image data in terms of the Wigner-Ville distribution (WVD). The choice of WVD is due to its optimum informationpreserving property, and it results in the least amount of spread in the time frequency plane. The detection is then carried out by using the PCA to extract the features and MPL networks to classify the patterns. However, the method can be performed with the maximum SNR at -25dB.

Last, In terms of circuit implementations, Spectrometer was introduced for weak signal detection by using nuclear magnetic resonance (NMR). The NMR absorption line in a differential form is detected by a repetitive oscillator frequency sweep and requires the static magnetic field to be run at a fixed value in the duration of the multiple pass experiment, i.e. for a period of days, by the reason of the signal-to-noise ratio is quite small and signal averaging is necessary. The oscillation level has to remain constant during the sweeps and large sweep amplitudes may be employed even at low oscillation levels. Since the spin-lattice relaxation times in most solids are very long, the low levels of radio frequency excitation must be maintained for the NMR in order to avoid saturation of the specimen while sweeping through the resonance region. The shape of a resonance line can be disturbed due to the instabilities of both the frequency and amplitude of the radio frequency field. A new solution in constructing the CW NMR measuring apparatus, by involving the using of the computer controlled PLL frequency-sweep NMR oscillator. The oscillator circuit of the spectrometer is considerably less sensitive to unwanted noise nuclear susceptibility component even in case of a very small signal-to-noise ratio, when the signals averaging is necessary. Nonetheless, Spectrometer has not widely been utilized due to the large circuit implementation, complicate techniques, and also high application cost.

In recent years, the principle of this WSD method based on chaotic oscillators was proposed as a new method for detection. A system parameter of the Duffing oscillator has a threshold. Around the threshold there is a state of conversion from the chaotic to the periodic and a monotonic functional relation between the system parameters and the characteristic of parameters which can be described the system state. Furthermore, it has been seen no effect with the strong background of noise which is namely ‘immunity’ to noises. In measurement, the system parameters will be adjusted to the threshold in order to process the system as the critical state. Consequently, the weak periodic signal is driven into the system as small perturbations through the system amplitude of a driving parameter. The parameter can be measured by the function relation. WSD method based on Duffing oscillator contains six parts which are detection task, detection system, detection conditions, chaos criterion, method improvements and method realizations. The Detection task is to measure parameters of the periodic signal to be detected. Such parameters include amplitude, phase and frequency. The periodic signal types include harmonic wave signal, square wave signal, triangle wave signal and short-impulse signals. The amplitude to be measured input is as small perturbation to system parameter.In the detection system, chaotic oscillator whose equation form is different would have different detection performance. In the additional forcing terms, the detection performance is determined by the form of non-linear terms in chaotic oscillator equation. Signals contain the different frequencies can be detected by transform '$t = \omega\tau$' where is a time in second, ω is the angular frequency in rad/second and τ is time-scaling parameter on the equation in time scaling terms.The detection condition includes noise condition and initial condition where noise condition means noise type and SNR.

The WSD method based on chaotic oscillator generally requires noise are zero-average, additive, stationary, Gaussian or 'white', the noise sometimes can also be non-Gaussian or 'color' noise. Color noise can be obtained as output by inputting white noise to the fourth-order band pass filter. As an index of detection performance, the SNR is generally required less than -10dB. Experiment method and method of statistically analyzing for the oscillator equation as stochastic differential equation. Chaos criterion is about identifying and describing state of chaos system. Chaotic state can be identified by phase trajectory approach which is based on analyzing phase trajectory as output of chaos system. It contains approaches of visual identification by phase figure, time series analysis, Poincare' section and power spectra analysis. The threshold of Duffing oscillators are determined analytically by Melnikov method. The range of chaos belts was theoretically predicted with different parameters, which is proved by the results of numerical simulations. The rule of the influence of systemic parameters (including amplitude and frequency of the external excitations, initial condition) to system behavior is deduced from a number of the experiments. In the realization method, Hardware realization of WSD method based on chaos oscillator

Table 1.1 Summary of particularly related techniques of weak signal detection methods using chaotic Duffing oscillators

Refs.	**Proposed WSD Methods**	**Advantages**	**Method Weakness**
[1]	Stochastic Resonance (SR)	Noise tuning.	Require more additional filter
[2]	Detection algorithm base on Wigner-Ville Distribution	Detection can be perform for multi-frequency	Data loss. Low SNR
[3]	Spectrometer	sensitive to unwanted noise	the large circuit and high application cost
[4]	Duffing Chaotic Oscillators	immunity to noise, multi-frequency and detected signal	Variation in system parameters and Nocircuit implementation

includes chaos measurement circuit and sometimes DSP is used. Method improvement, WSD method based on chaos oscillator can be combined with traditional WSD method such as correlation method, cross spectra method, Boxcar Intergraph and lock-in amplifier (LIA).

Since there are numbers of experiments lead to chaotic Duffing oscillators. However, the study is limited to theory, method, simulation experiment condition and result. The experiment has no concrete process and cannot be repeated and proved by researchers. The corresponding application and effect are unknown. Chaos detection model used in the experiment and the phenomena are not specifically analyzed with the chaos theory leading to a low number of technology implementation innovations. In practical, orderly and universal conclusion is not concluded. The realization of quantitative method and system precision validity and characteristic are seldom studied. The author has observed such a number of researches in chaotic Duffing oscillator gap leading to the thesis motivation.

1.2 Motivations

Weak signal detection is a challenging topic in signal detection field. There are number of major motivations that have led the author to the research and development in this thesis. Due to the high demand in mechanical industry measurement, weak signal detection has been employed as a tool to detect an Acoustic Emission (AE) in order to exam the corrosion of cutting tools in metal cutting process ,a corrosion in reinforced concrete structures and in the fuel pipes line. Especially, for recently problem in advanced applications of AE technology with high-cost and the imperfections of the technical terms such as the buried of noise in the detected signal.

In chaos scheme many studies only solve a problem in a special measurement condition with random situations, and the complete measurement method system and its theory base have not been developed, The experiment conditions are too ideal without noise influence for example, which make the results and effect very good and the method is unusable in detection. Inspiring the author to develop the potential application which immune to noise and sensitive to weak signal emission and extract the underlying (high-accuracy) model in Weak Signal Detections in all involving parameters, research in the combination from chaos theory until the hardware implementation and also generalize the form of Duffing chaotic oscillator.

1.3 Statement of Problems and Hypothesis

Recently, the chaotic oscillator has been proposed and investigated for detecting weak signal in the presence of the strong noise. The precision of the system depend on the parameter threshold value still unsolved. Even a small variation of parameter value may cause a dramatically change in the chaotic system behavior. It has been the curiosity to extract the underlying model in Weak Signal Detection and simplify the generalization forms of Chaotic Duffing Oscillator in Weak Signal Detection with high-accuracy application circuit may lead the better result. There is also no circuit implementation using chaotic Duffing oscillator.

1.4 Objectives

1.4.1To perform the mathematical analysis and parameter optimizations of Duffing oscillator.

1.4.2To design the weak signal detection methods with the chaotic Duffing oscillator with high accuracy and low SNR.

1.4.3 To implement the electronic WSD circuit and system using chaotic oscillator.

1.5 Research Scopes

This thesis has been researched in three major parts which consisting of the Chaos theory analysis, system simulations and circuit implementation.

1.5.1 Study in chaos theory, dynamical system, and nonlinear analysis through the mathematical equation system such as dynamic equation, time-scaling model, Eigen value, Eigen-Vector, Jacobian-Matrix and also Stability analysis. Study in chaotic indicators such as attractor, time-domain, Poincare' section, Bifurcation, Lyapunov diagram and Kaplan-York dimension. Thesis also integrates the model of chaotic Duffing oscillator by generalize the form of the Duffing oscillator equation.

1.5.2 Study the method to determine the chaotic Duffing Oscillator systems through the parametric excitation method and additional forcing terms. Optimize the entire parameters in Duffing equation of damping ratio, Small driving amplitude (0-1V.) and also of the parameters deviations. Characterize the dynamic behaviors and bifurcation boundaries through the use of a positive Lyapunov diagram and a dimensionless Kaplan–Yorke dimension (D_{KY}) in Matlab simulation program.

1.5.3 Implement the Chaotic Duffing circuit by using circuit board and electronic complex devices.

1.6 Expected Outcomes

1.6.1 To gain the knowledge of Chaos theory and dynamical systems.

1.6.2 To gain the knowledge of Weak signal detection methods using Duffing chaotic oscillator.

1.6.3 To perform the mathematical analysis and parameter optimizations of Duffing oscillator.

1.6.4 To generate the weak signal detection methods with the chaotic Duffing oscillator with high accuracy and low SNR.

1.6.5 To create the electronic WSD circuit and system using chaotic oscillator.

Chapter 2
Literature Reviews

This chapter describes a number of the related theories and experimental result of weak signal detection and measurement such as Chaos theory and dynamical system, an existing of differential equations with chaotic behaviors including Lorenz systems, Rössler systems and Chua's systems, Chaos Theory and Weak Signal Detection in the six related methods, Weak Signal Detection Method Base on Duffing Chaotic Oscillator and also the circuit implementations.

2.1 Chaos Theory and Dynamical Systems

Chaos is a term to describe a behavior of dynamical systems that appears closely to random, underlying with mathematical order in the dynamic system. Chaos is normally occurs in nature, however, the chaotic characteristic still confuse with the random behavior. Chaos can occur only in nonlinear systems and characterized by a breakdown of predictability known as sensitive dependence on initial conditions which is the most important distinguishing feature of chaos. This implies that even though chaotic systems are deterministic (unlike systems exhibiting random behavior), even the smallest difference in initial state can cause a dramatically difference in the final state. The chaotic behavior generally described as "the butterfly effect" where the flapping of a butterfly's wings in may lead to a radical change in weather in the difference side of the world. Long term predictability of chaotic systems is impossible since all numerical calculations have a finite non-zero error which will diverge over time and the predictions unreliable. The chaotic behavior contain three majors properties , Chaos can occur only in deterministic nonlinear dynamical systems, Chaotic behavior looks complicated and irregular but has an infinite number of unstable periodic patterns embedded in the system and Chaotic behavior is sensitive to initial conditions

Dynamical systems are the study of the long-term behavior of evolving systems. The modern theory of dynamical systems originated at the end of the 19th century with fundamental questions concerning with the stability and evolution of the solar system. Attempts to answer those questions led to the development of a rich a powerful field with applications to physics, biology, meteorology, astronomy, economics, and other areas. By analogy with celestial mechanics, the evolution of a particular state of a dynamical system is referred to as an orbit. A number of themes appear repeatedly in the study of dynamical systems such as properties of individual orbits, periodic orbits, typical behavior of orbits, statistical properties of orbits, randomness vs. determinism, entropy, chaotic behavior and stability under perturbation of individual orbits and patterns. Normally, the dynamical systems can be written in the system of mathematically equations that describe how each variable changes with time as follows;

$$\begin{aligned} \frac{dx_1}{dt} &= f_1(x_1, x_2, x_3, \ldots, x_n, t) \\ \frac{dx_2}{dt} &= f_2(x_1, x_2, x_3, \ldots, x_n, t) \\ &\vdots \\ \frac{dx_n}{dt} &= f_n(x_1, x_2, x_3, \ldots, x_n, t) \end{aligned} \tag{2.1}$$

Where n species are given by $(x_1 \ldots, x_n)$ and the right side of each equation is a function of $(f_1 \ldots, f_n)$ that indicated the variables changes with time.

2.2 Differential Equations with Chaotic Behaviors

2.2.1 Lorenz System

In 1961, Lorenz has visited to Barry Saltzman of the Travelers Insurance Company Weather Center in Hartford, Connecticut, Lorenz was shown a seven-equation model of convective motion in a fluid heated from below and cooled from above. Saltzman’s seven equations were themselves the reduction from a set of partial differential equations describing Rayleigh-B´enard convection, which the Saltzman’s equation study how heat rises through a fluid like air or water and reduced the system into three equations as follows;

$$\dot{x} = -\sigma x + \sigma y$$
$$\dot{y} = -xz + rx - y \qquad (2.2)$$
$$\dot{z} = xy - bz$$

The model is highly idealized of a fluid, the warm fluid below rises and the cool fluid above sinks, setting up a clockwise or counterclockwise current. The Prandtlnumber σ, the Rayleigh (or Reynolds) number r and r, and b are parameters of the system. The variable x is proportional to the circulatory fluid flow velocity. If x >0 , the fluid circulates clockwise while x<0 means counterclockwise flow. The variable y is proportional to the temperature difference between ascending and descending fluid elements, and z is proportional to the distortion of the vertical temperature profile from its equilibrium (which is linear with height). For setting $\sigma = 10, b = 8/3$ Lorenz found numerically that the system behaves "chaotically" whenever the Rayleigh number r exceeds a critical value. The solutions appear to be sensitive to initial conditions, and almost all of them are apparently neither periodic solutions nor convergent to periodic solutions or equilibria.

2.2.2 Rössler System

The Lorenz systems have been studied in detail because it can be seen a treasure trove of an interesting phenomena. The system was the widely known chaotic attractor from a set of differential equations. The dynamical equations are simple; the various types of dynamical behavior can be seen for different parameter ranges. Subsequently, the others chaotic systems of differential equations have been identified such as Rössler systems, Chua circuits etc. In 1976, the German Scientist O. Rössler found a system which create a chaotic attractor with an even simpler set of nonlinear differential equations name "The Rössler systems" as follows;

$$\dot{x} = -y - z$$
$$\dot{y} = x + ay \qquad (2.3)$$
$$\dot{z} = b + (x - c)z$$

For the criterion of parameters a= 0.1, b= 0.1, and c =14, there is an apparent chaotic attractor, The Lyapunov exponents for this attractor have been measured by computational simulation to be approximately 0.072, 0 and -13.79. The corresponding

Lyapunov dimension is 2.005. Rössler primarily considered a slightly different set of parameters, a =0.2, b =0.2, and c =5.7, but the properties of the systems are not dramatically different for these values.

2.2.3 Chua's System

In the 1980s, Leon Chua and colleagues developed a class of electronic circuit capable of exhibiting chaos with a wide range of behaviors. The system has been extensively studied by many researchers. The systems contain an RLC circuit with four linear elements (two capacitors, one resistor, and one inductor) and a nonlinear diode, which can be modeled by a system of three differential equations as follows;

$$\begin{aligned} \dot{x} &= ay - x + |x+1| - |x-1| \\ \dot{y} &= z - x \\ \dot{z} &= y \end{aligned} \tag{2.4}$$

where*a* is the systems parameter and *x, y, z* are the systems variables. Chua circuits was widely utilized and continuously modified by a number of researchers in nonlinear terms.

2.3 Chaos Theory and Weak Signal Detection

Chaos theory for the detection of weak signals derived from the discovery of chaos in nonlinear dynamic system. The research of chaos theory indicates that certain nonlinear chaos systems have the sensitivity to small signal and the immunity to noise under the controlled condition, utilizing this characteristic, takes detecting signal as the driving force of the chaos system, although the noise is intense, it has no influence on system mode's change, however once has specific signal, even if signals amplitude is quite small, also make the system change. The computer recognizes system modes by the method of image recognition or envelope abstraction and so on, and then judges whether signal is exist, thus achieve the goal of detecting weak signal from strong background noise. The extreme sensitivity of the chaotic dynamic system behavior for the initial parameters results in to study it on weak signal detection. Especially, because of clear physical meaning and convenient debugging of Duffing oscillator, it became the source of weak signal detection research.

2.4 Weak Signal Detection Based on Duffing Chaotic Oscillator

Weak Signal Detection (WSD) techniques have been utilized extensively for the detection of weak periodic signals under strong additive noises, i.e. low SNR [5], for various applications such as GPS signal detection [6], tool wear detection [7], down hole acoustic telemetry system [8], and EEG analysis [9]. Existing WSD techniques include, for example, a periodic signal relevant detection [10], a periodic signal-sampling-points integration [11], a single frequency-locked detection [12], and a linear filtering method [13]. However, these WSD techniques are relatively complicated; exploiting specific signal processing such as Fourier or Wavelet transforms in order to extract the desired signal embedded in strong noise circumstances, and also requiring complicated implementations of circuit and systems. The WSD technique based on chaotic systems has been suggested as an alternative to those existing WSD techniques due to the capability of detecting sensible signals and the immunity to noises under specifically controlled conditions. The chaotic system is typically a deterministic nonlinear dynamic system, which is highly sensitive to initial conditions and possesses unpredictable long-term behaviors [14]. Specifically, the class of non-autonomous chaotic systems offers high suitability for WSD since the dynamic behaviors depend on a periodic stimulus, which can be realized as a signal desired to be detected.

In particular, Duffing oscillator has been employed considerably as the forcing signal can be detected efficiently although its amplitude is small. Two early approaches for realizing a detecting signal in Duffing oscillator include (1) the parametric excitation and (2) the addition of forcing terms. On the one hand, the parametric excitation approach uses an external signal desired to be detected as the forcing signal in the typical Duffing oscillator [15-16]. In the case where the parameters of an external signal satisfy the criteria of system dynamics, a chaotic attractor will exist apparently and the desired signal can therefore be detected by observing the existence of this attractor. On the other hand, the addition of forcing term approach primarily determines one internal forcing signal and its parameters are also set in such a way that the system dynamics is readily in a chaotic state, providing an apparent chaotic attractor [17]. The external weak signal desired to be detected is subsequently added to the internal forcing signal, leading to the transition between periodic and chaotic states. The detection can be made by observing the changing aspects of an attractor. Recently, Melnikov [18] and Lyapunov [19] functions have been suggested in addition to those two approaches, which are mainly based on human observation on chaotic attractors. The Melnikov function provides a numerical threshold value from a Melnikov function plot in order to monitor chaos criteria, but the detection is still made by human observation. The Lyapunov function offers a quantitative measure of chaoticity through the positive Lyapunov Exponent (LE), avoiding human observation and improving the efficiency. Nonetheless, the positive LE depends on system parameters and can be designed as large as required through a linear time rescaling that has no effect on system dynamics [20]. The characterization and optimization of all system parameter are therefore necessary.

Table 2.1 Summary of particularly related techniques of weak signal detection methods using chaotic Duffing oscillators.

Ref.	Years	Techniques	Proposed Equations
[21]	2003	Recovery force terms	$\ddot{x}+k\dot{x}-x^{3}+[1+aS_{T}(t)]x^{5}=\gamma\sin(\omega t)$
[22]	2005	LE Thresholds method	$\ddot{x}+\delta\dot{x}-x+x^{3}=f\cos(t)+Z(s)$
[23]	2007	Effect of Amplitude Modulation(AM) by Bifurcation	$\dot{x}=y$ $\dot{y}=\omega_{0}^{2}x-\beta x^{3}+\varepsilon[-\alpha\dot{x}+(f+2g\cos\omega t)\sin\Omega t]$
[24]	2010	Melnikov Function, Phase-Shift and Frequency Control	$\frac{1}{\omega^{2}}\ddot{x}_{\tau}(\tau)+\frac{\delta}{\omega}\dot{x}_{\tau}(\tau)-x_{\tau}(\tau)+x_{\tau}^{3}(\tau)=F\cos(\omega\tau)$
[25]	2011	Wavelet Denoising Method	$\dot{x}=\omega y$ $\dot{y}=\omega(-ky+x-x^{3}+f\cos(\omega\tau))$
[26]	2011	Frequency Spectrum	$\ddot{x}+0.5\dot{x}-x+x^{3}=0.825\cos(\omega\tau)+0.001\cos(\omega\tau)$

As shown in Table 1, Li Yueand Yang Baojun (2003) [21] have early proposedachaotic system for the detection of periodic signals under the background of strong noise and also proposed a new method to study the chaotic system for the detection of periodic signals in the presence of the strong background of noise through the modification of Duffing-Holmes equation.

$$\ddot{x} + \delta\dot{x} - x + x^3 = \gamma \sin(\omega t) \tag{2.5}$$

The author has modified the equation (1) by setting a periodic driving force as an invariable and transform the recovery force term to $-x^3 + cx^5$ where $c = 1 + aS_T(t), a \geq 0$ The equation can be transformed as follows;

$$\ddot{x} + k\dot{x} - x^3 + [1 + aS_T(t)]x^5 = \gamma \sin(\omega t) \tag{2.6}$$

The result of the numerical experiments indicated that the chaotic system of the equation (2.6) is sensitive to the weak periodic signal mixed with a perturbation noise. The signal to noise ratio for the system can reach to about – 91 dB.

Chongsheng Li (2005) [22] has proposed a new weak signal detection methods based on Lyapunov Exponents (LE) by a model based on LE calculation and an improvement of LE calculation algorithm. On the one hand, the model based on LE calculation is used to determine the specification of the initial threshold value. On the other hand, the improvement of LE calculation algorithm is proposed based on phase space reconstruction of the observed data. The characteristics of the improvement methods have high precision of the LE threshold value and the automatic recognition. The simulation results verify the analysis and the effectiveness of this method. The proposed equation can be obtains as follows.

$$\ddot{x} + \delta\dot{x} - x + x^3 = f\cos(t) + Z(s) \tag{2.7}$$

The equation (2.7) can be analyzed by LE method. The analysis and simulation results show the chaotic characteristic criterion based on LE method has an advantage of high accuracy and easy implementation for setting the LE threshold value for adjust the periodic driving amplitude.

Ravichandrana , Chinnathambi and Rajasekar (2007) [23] have proposed the homoclinic bifurcation and its transition from regular to asymptotic chaos in Duffing oscillator subjected to an amplitude modulated force in both analysis and numerical experiment. Applying the Melnikov analytical method, the threshold condition for the occurrence of horseshoe chaos is obtained. The Melnikov threshold curves are drawn in different external parameters space. Analytical predictions are demonstrated through the direct numerical investigations. The Parametric regimes where suppression of horseshoe chaos occurs are predicted. Period doubling route to chaos intermittency route to chaos and quasi-periodic route to chaos are found to occur due to the amplitude-modulated force. The state equation can be show as follows;

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= \omega_0{}^2 x - \beta x^3 + \varepsilon[-\alpha\dot{x} + (f + 2g\cos\omega t)\sin\Omega t] \end{aligned} \tag{2.8}$$

The numerical investigations show the stable and unstable manifolds of saddle, maximal Lyapunov exponent, Poincare´ map and bifurcation diagrams are used to detect homoclinic bifurcation, the effect of the external force on horseshoe chaos and routes to asymptotic chaos. Melnikov analytical method was utilized in order to obtain the threshold condition for onset of horseshoe chaos that is transverse intersections of stable and unstable branches of homoclinic orbits. Threshold curves are drawn on different parameters space. The author verified the analytical predictions through numerical simulation. The paper demonstrated the effect of the parameters *f*, *g* and Ω on the dynamics of the system. The homoclinic orbits of Duffing oscillator are driven by the sinusoidal force, $f\sin(\omega t)$ or $f\cos(\omega t)$, exhibited one of the two possible behaviors depending upon the control parameters, The stable and unstable branches of homoclinic orbits are well separated and the Transverse intersections of stable and unstable branches of both W^+ and W^-.In the case of the sinusoidal force is replaced by the AM force in addition to the possible behaviors the system shows additional features: transverse intersections of stable and unstable portions of W^+ alone and W^-alone. These two additional possibilities occur for the range of control parameters. Such range of parameters predicted by Melnikov method is numerically verified. The amplitude modulated force considered in the present work has four parameters. The presence of additional parameters can be used to control and anti-control of chaos.

Jian-xiong and ChulinHouwang (2010) [24] have proposed the Melnikov function to determine the threshold of system parameters in Duffing oscillator. The dynamical change of the system can be analyzed by the phase transitions in phase space diagram. Through analyzing the intermittent chaos mechanism of Duffing oscillator, the system output is a intermittent chaotic signal when input frequency deviates from the compulsory drive frequency slightly. The frequency deviation can be estimated by the statistic of output chaotic signal. The experiment results indicated that the weak signal detection method based on Duffing oscillator can detect weak sinusoidal signals with extremely low SNR and frequency to be detected. The author proposed an equation as follows;

$$\frac{1}{\omega^2}\ddot{x}_\tau(\tau)+\frac{\delta}{\omega}\dot{x}_\tau(\tau)-x_\tau(\tau)+x_\tau{}^3(\tau)=F\cos(\omega\tau) \qquad (2.9)$$

Yusheng Sun et al. [25] (2011) have proposed a method of Weak signal detection based on Duffing oscillator by wavelet denoising method. The Simulation experiment shows that using joint measurement system to detect weak signal from high background of noise have more satisfy result. The author proposed the ability of detecting weak signal by using follows equations.

$$\begin{aligned}\dot{x}&=\omega y\\ \dot{y}&=\omega(-ky+x-x^3+f\cos(\omega\tau))\end{aligned} \qquad (2.10)$$

The method can detect weak signal in the low SNR with a high precision and strong ability to adapt. The author suggested the method is simple, intuitive and easy to implement. As shown in table 1, AbolfazlJalilvand and HadiFotoohabadi (2011) [26] have proposed a method for identifying the chaotic state of the Duffing oscillator based on frequency spectrum analysis. The proposed method has three properties reasonable calculation of complexity, robustness to moderate noise amount and capability of detection with short signal sequence. The author proposed the equation as follows;

$$\ddot{x}+0.5\dot{x}-x+x^3=0.825\cos(\omega\tau)+0.001\cos(\omega\tau) \qquad (2.11)$$

Table 2.2 Summary of particularly related circuit of weak signal detectionbased on circuit improvement

Refs.	Proposed Circuits	Advantages	Disadvantages
[27]	Signal correlation	Simple and direct implantation	Large circuits with two processing signals
[28]	Duffing Oscillator	Good Small-amplitude signal detection	Complicated circuit
[29]	Duffing Oscillators and control circuit	Simpler circuit	Vulnerable to components

2.5 Circuit Implementations

The conventional weak signal detection method is the transformation of non-power signals to the power signals by the conversion circuits following with amplifying and filtering the transformed signals. The method is appropriate to the efficiently distinguishable output signals from the circuits, but in competent to the weak signals which frequency is low and also weak amplitude. Apparently, weak signals are usual the signals immense to the background of noises and the driving amplitude are relatively weak compared with the noises embedded amplitude .therefore, an efficient method is essential for the weak signal detection in information dealing, method of weak signal detection based on the signal correlation principle was introduced. Table 3 shows the summary of particularly related circuit of weak signal detection based on circuit improvement. In [27], the method principle consist of two stage parts, Signal modulation stage and a detection stage, the signal modulation is the certain characteristics of measured signals and a reference signal. In order to obtain effective results of phase-sensitive detection, the reference signal should meet the condition of its frequency is 20 times of the measured signals $(\omega_{reference} >> \omega_{signal})$. Thefrequency spectrum of the output signals is a frequency band between

$\omega_{ref.} - \omega_{signal}$ and $\omega_{ref} + \omega_{signal}$ from the modulated equation where $V_m$is the modulated signal.

$$V_m(t) = 0.5\cos(\omega_{ref} + \omega_{signal})t + 0.5\cos(\omega_{ref} - \omega_{signal})t \tag{2.12}$$

The demodulation procedure is realized by the phase sensitive detector. The essence is that the modulation signal is multiplied by the reference signal again where V_d is the demodulated signal.

$$V_d(t) = AV_m(t)\cos\omega_r t \tag{2.13}$$

This method can be implemented through relatively complicated circuits as shown in Figures 2.1 and 2.2.

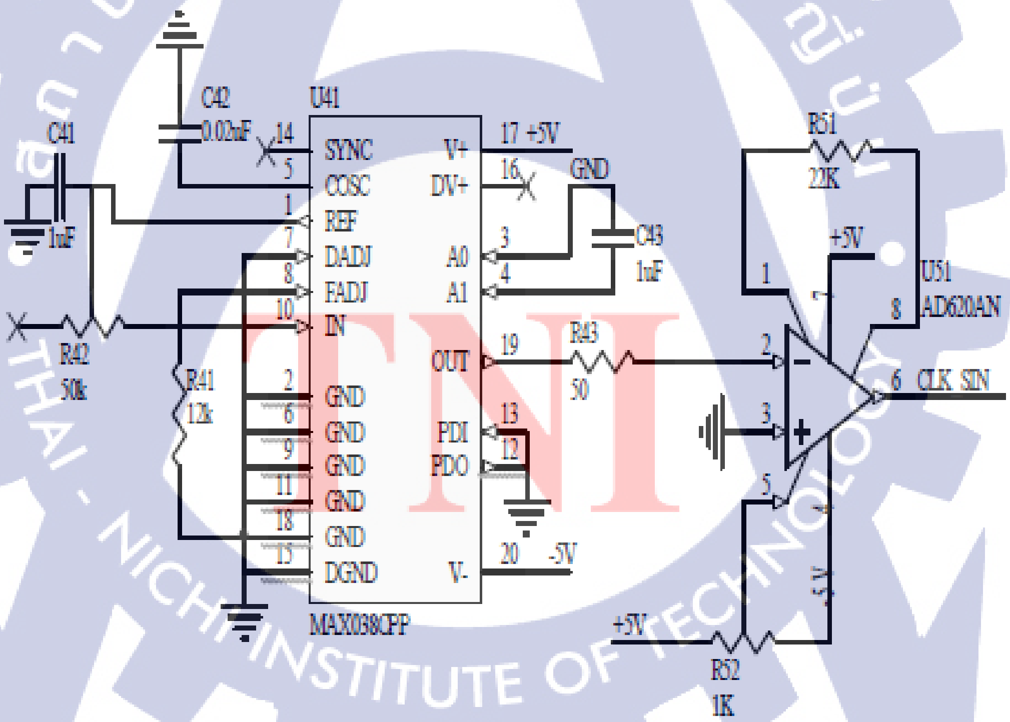


Figure 2.1 The circuit diagram of signal generator using microcontroller [27].

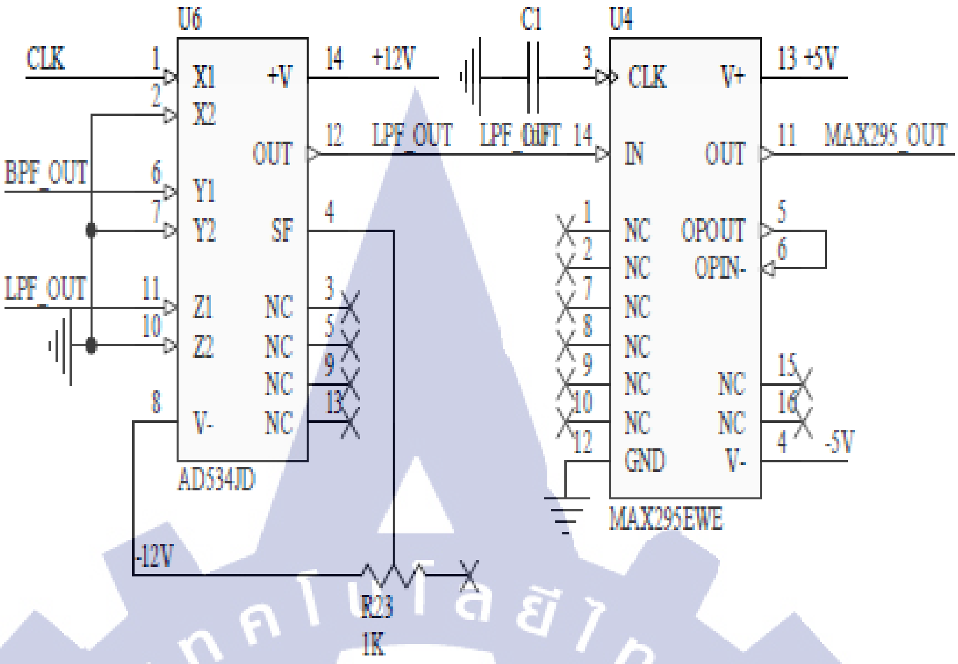


Figure 2.2 The circuit diagram ofa phase sensitive detector [27].

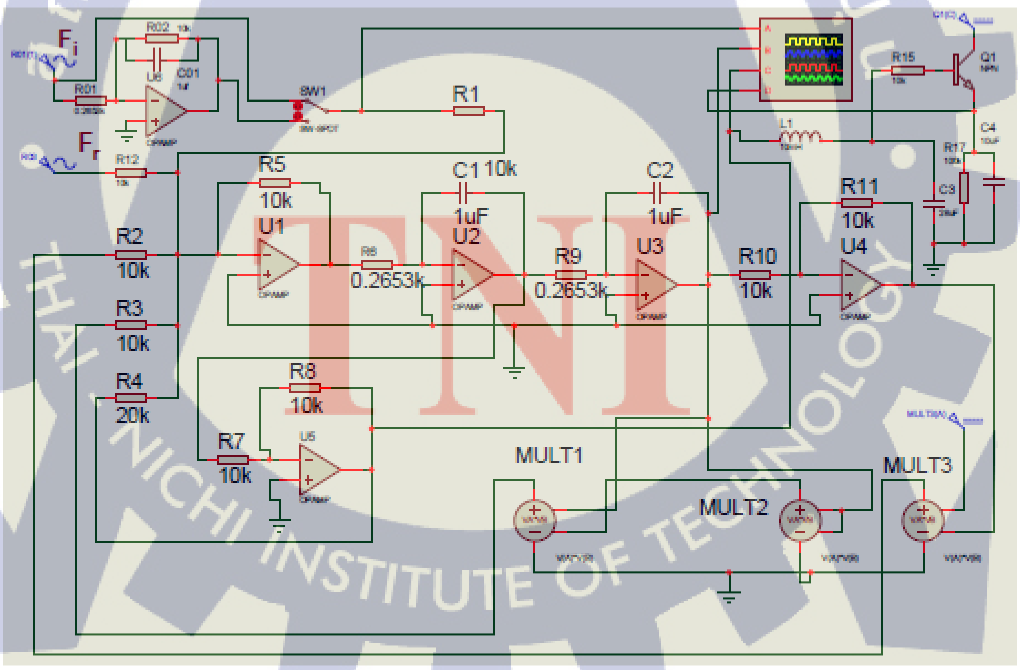


Figure 2.3 Circuit diagram of the Duffing oscillator suing multiple integrators [28].

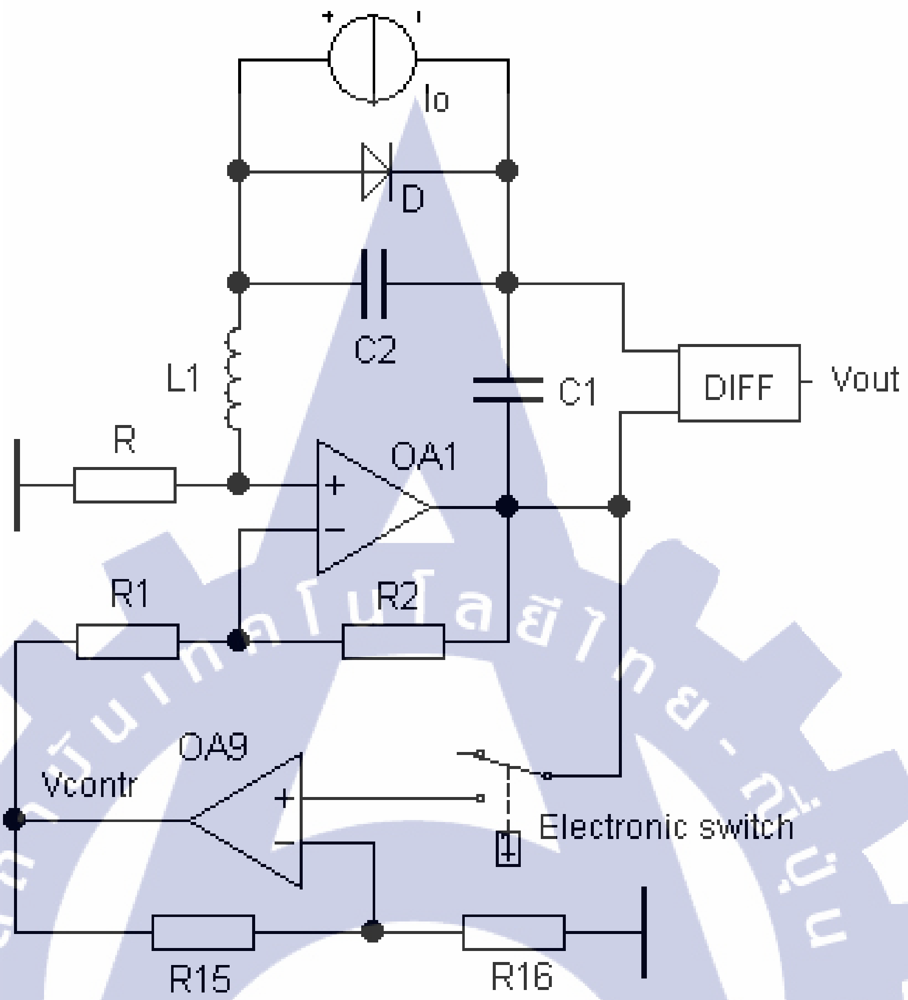


Figure 2.4 Circuit diagrams of the chaotic oscillator and electronic switch [29].

The circuits designed based on the signal correlation principle distinguishes to the weak signal from the noises efficiently. Nonetheless, due to the large circuit lead user to high application cost.The traditional weak signal detection mainly uses the method of linear filtering and signal superimposition to extract the signal. However, it can be seen that such methods often unable to detected signal when the background noise is strong and the detecting signal is weak, cannot meet the needs of weak signal detection requirements. Although modern detection methods have apparently effect, strong adaptability, but most of them have complex structure, difficult to achieve. Therefore, the weak signal detection technology has a new breakthrough when the chaos theory is introduced in the signal detection. The Chaos detection method is different from a variety of existing measuring methods, which is a new signal processing method.Duffing oscillator simulation circuit has been introduce through sinusoidal voltage source, analog operational amplifier, analog multiplier, resistors and capacitors as shown showing in Fig3.all components are the ideal virtual electronic components. According to the basic feature of an ideal operational amplifier, system could easily derive the circuit equation of Duffing chaotic system as follows in Figures 2.3 and 2.4.

In conclusion, WSD has been extensively utilized based on chaos theory and dynamical systems. Due to WSD method based on chaos oscillator combined with traditional WSD method such as the Recovery forcing terms methods, the LE Thresholds method, the Effect of Amplitude Modulation (AM) by Bifurcation and Melnikov Function, the Phase-Shift and Frequency control and Wavelet Denoising Method. Such methods were proposed based on chaotic Duffing oscillators can be enhanced by complex numerical function through high-processing software. However, an existing algorithm and methods can be analyzed and further improve through the equation modification and parameters variation will be performed in the following chapters.

Chapter 3
Research Methodology

3.1 Introduction

The research in this thesis is to enhance the original existing stock of knowledge about chaotic theory and making for its advancement. The researches in this methodology include the study of theories, observation, comparison, analyzed and experimentation. The research for knowledge in weak signal detection base on the Duffing oscillator can be perform through objective and systematic method of finding solution to research gap techniques. The systematic approach concerning generalization and the formulation of chaos theory until the circuit implementation.

3.2 Overall Research Process

The research will mainly study in 5 directions consisting of

3.2.1 To study Chaos theory and dynamical systems.

3.2.2 Weak signal detection methods using Duffing chaotic oscillators.

3.2.3 To perform the mathematical analysis and parameter optimizations.

3.2.4 To generalize the form of Chaotic Duffing oscillator.

3.2.5 To create the electronic WSD circuit using Chaotic Duffing oscillator

3.3 Utilizing Data

To generalize the form of the chaotic Duffing oscillator, the simulated data in the parametric excitation include;

3.3.1 δ : The damping ratio varied from zero to unity.

3.3.2 γ : The periodic driving force.

3.3.3 ω : The frequency of driving force.

3.3.4 $\Delta\gamma$: The periodic driving force deviations.

3.3.5 $\Delta\omega$: The frequency of driving force deviations.

3.3.6 Additional noise terms n(t)

3.4 Research Tools

In the thesis, the entire of simulation results were perform through Matlab® version R2008a, Pspice Orcad student version, and Labview 2009.

3.5 Data Analysis Methods

The numerical analysis of chaotic theory such as Eigen-value and Jacobian matrix which specify the characteristic of the chaotic behavior in phase space diagram. Subsequently, The mathematic simulation function were analyzed though Matlab® version R2008 program as an indicating tools for detect the chaotic behavior such as positive Lyapunov (LE^{+}) indicate the systems is in chaotic state where D_{KY}>2 and complex in bifurcation diagram can be also obtain in chaotic state. Circuit design is performed through Pspice and verifications will exploit the Labview for noise generation and measurements.

3.6 Research Procedures

3.6.1 Analyze the Duffing chaotic model through the dynamic system by using time-scaling method, Eigen value, Eigen-Vector, Jacobian-Matrix and Stability analysis.

3.6.2 Use the numerical result from 6.1 to analyzed by chaotic indicators such as attractor, time-domain, Poincare' section, Bifurcation, Lyapunov diagram and Kaplan-York dimension.

3.6.3 Optimize the system parameters in 6.2. And the research will also enhance the model of chaotic Duffing oscillator by generalize the form of the Duffing oscillator equation.

3.6.4 Implement the 6.3 circuit and demonstrate the Duffing chaotic Oscillator application in the constructed Circuit.

Chapter 4
Simulations and Experimental Results

This chapter deals with the investigations, simulations and experimental results of Duffing chaotic oscillator. First, dynamic behaviors of Duffing chaotic oscillator is investigated through Time Domain Analysis, Chaotic Attractors, Bifurcation Diagrams, Lyapunov and Kaplan-Yorke Dimensions and Poincare section. Second, optimizations of Duffing Chaotic Oscillators Based on *LE* and D_{KY} are described as a new method in finding the robustness of the chaotic system prior to the use in weak signal detection. Last, proposed weak signal detection methods are described, including phase-plane investigation, and circuit designs using discrete components. This chapter summarizes the resulting works of this thesis involving all theoretical matters and experimental verifications.

4.1 Investigations of Duffing Chaotic Oscillator Dynamics

Based on general form of Duffing's Equation which is described by a second-order non-autonomous jerk equation as

$$\ddot{x} + \delta\dot{x} + \frac{dV(x)}{dt} = f(t) \tag{4.1}$$

where x is a state variable, δ is a damping ratio, and $V(x)$ is a nonlinear restoring force function. The driving signal $f(t)$ is a time-varying periodical cosine signal, i.e. $f(t) = \gamma\cos(\omega t)$ where γ and ω are an amplitude and an angular frequency, respectively. Typically, the most common nonlinear function is given by $V(x) = 0.5\alpha x^2 + 0.25\beta x^4$ where α and β are constants. Such a function $V(x)$ is a double-well case that describes the motion of a classical particle in a double-well potential, and its solutions are always bounded by the strong cubic restoring force. Consequently, the general form of Duffing's Equation can alternatively be expressed as

$$\ddot{x} + \delta\dot{x} + \alpha x + \beta x^3 = \gamma\cos(\omega t) \tag{4.2}$$

This Equation (4.2) possesses rich dynamic behaviors based on the signs of α and β for appropriately chosen values. Generally, the absolute values of α and β are arbitrarily set to unity so that the local maximum of $V(x)$ is located at x=0. In addition, Equation (4.2) can exhibits chaos when α<0 and β>0 and the chaotic attractor is called Duffing's two-well oscillator. As a result, the particular form of the Duffing's equation is given by

$$\ddot{x} + \delta\dot{x} - x + x^3 = \gamma\cos(\omega t) \tag{4.3}$$

It can be considered from (4.3) that its dynamic behaviors ultimately depend upon only the parameter γ and ω of the driving signal under a specific value of damping ratio. Such an aspect is subsequently applicable to signal detection approach since the changes in γ and ω of the driving signal may lead to the changes in dynamic behaviors.

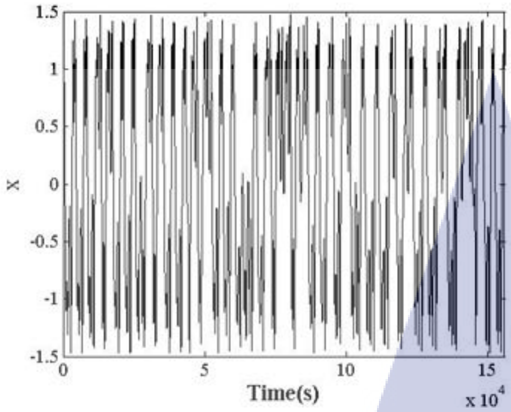
In order to detect the signal amplitude at any specific frequencies, Equation (4.3) can be normalized through time scaling method, i.e. $t=\omega\tau$, and therefore the particular form of Duffing's equation with frequency transition can be expressed as

$$\frac{\ddot{x}_\tau}{\omega^2} + \frac{\dot{x}_\tau}{\omega} - x_\tau + x_\tau^{\ 3} = \gamma\cos(\omega\tau) \tag{4.4}$$

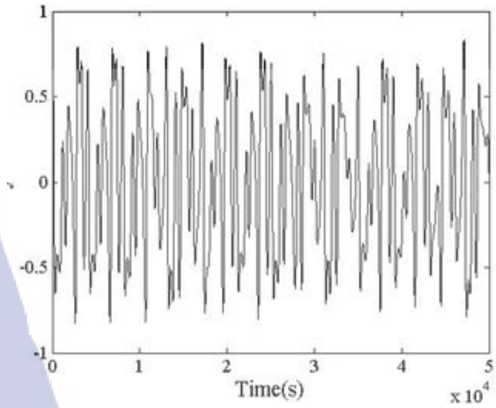
For the sake of simplicity, the subscript of x_τ is removed, and the dynamical form of (4.4) can consequently be expressed as

$$\begin{aligned} \dot{x} &= \omega(y) \\ \dot{y} &= \omega(-\delta y + x - x^3 + \gamma\cos(\omega t)) \end{aligned} \tag{4.5}$$

As a result, Equation (4.5) represents the particular form of Duffing equation with a wide range of dynamic behaviors for any frequencies under specific values of the forcing signal amplitude and the damping ratio.



(a) State variable x



(b) State variable y

Figure 4.1 Time–domain waveforms of state variable x and y.

4.1.1 Time Domain Analysis

Duffing Chaotic oscillator dynamics using the approximate solution methods of ordinary differential equations, *ode45*, named Runge–Kutta algorithm. Numerical simulations have been performed in MATLAB. The initial conditions of the systems were set to (0,-1.6,-1) with the difference values of δ = 0.34, γ = 0.37and with f = 10K Hz. Once the weak periodic signal is introduced to the system, the system instantaneously converse from the periodic state enters to the chaotic state. Figure 4.1 shows the time–domain waveforms of state variable x and y. It can be seen that both signals are random showing chaotic behaviors in time domain.

4.1.2 Chaotic Attractors

An attractor is a set towards which a variable, evolving according to the fixed point a dynamical systemover time. Those variables represent algebraically as an *n*-dimensional vector. The attractor is a region in *n*-dimensional space. It can be seen that, there have a various types of attractor such as, a finite set of points, a curve, a manifold, or even a complicated set with a fractal structure named as a strange attractor. The trajectory can be either periodic or chaotic state. The Duffing chaotic oscillator attractor shows stable limit cycles around two differentwellsFigure 4.2shows the simulation shows the various forming of Duffing chaotic oscillator attractor with perturbation of small sinusoidal signal with different

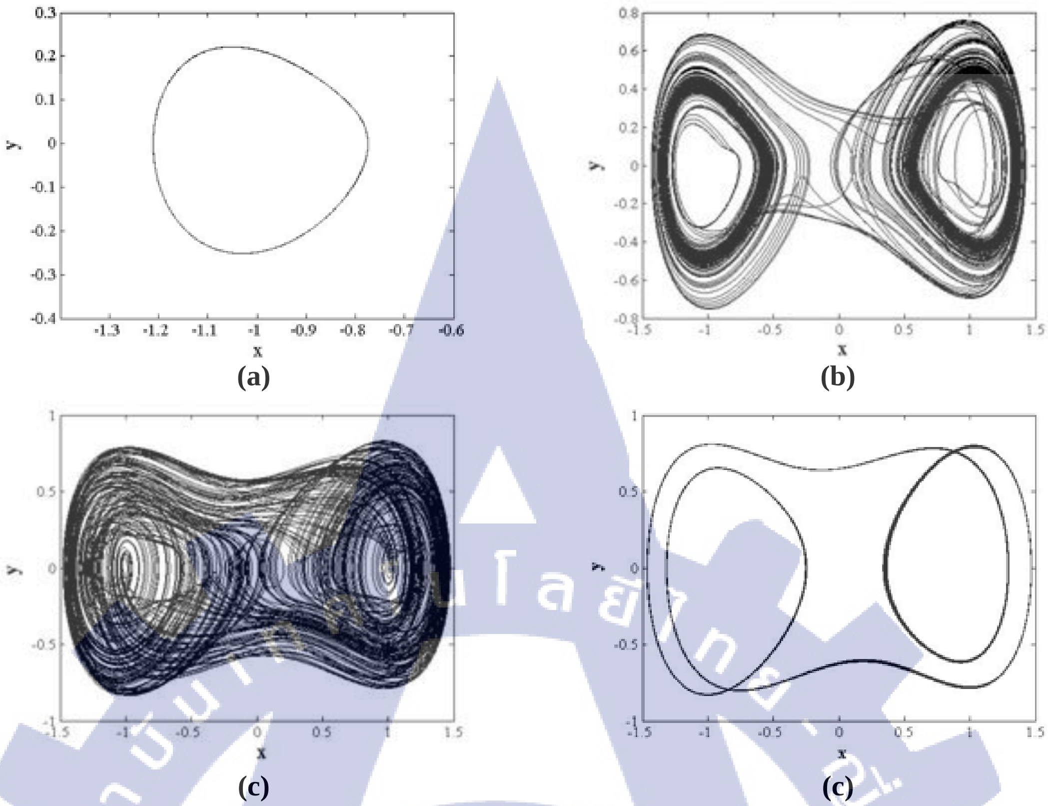


Figure 4.2 Dynamics forming of the Duffing chaotic attractor at δ = 0.34 and (a) γ =0.21(periodic),(b) γ =0.281(chaotic),(c) γ =0.37(chaotic) and ,(d) γ =0.41(periodic).

parameters when parameter δ was fixed. Figure 4.3shows the simulation shows the various forming of Duffing chaotic oscillator attractor with perturbation of small sinusoidal signal with different parameters when parameter γ was fixed.

4.1.3 Bifurcation Diagrams, Lyapunov and Kaplan-Yorke Dimensions

Numerical simulations have been performed in MATLAB using the initial condition of (x_0, y_0, z_0) = (0,-1.6,-1) In fact, the initial condition is not crucial, and can be selected from any point that lies in the basin of attractor. A bifurcation diagram shows the possible long-term values, i.e. fixed points or periodic orbits, of a system as a function of a bifurcation parameter in the system. It is usual to represent stable solutions with a solid line and unstable solutions with a dotted line.The chaoticity is a measure of the greatest *LE*, which is the average rate of growth of the distance

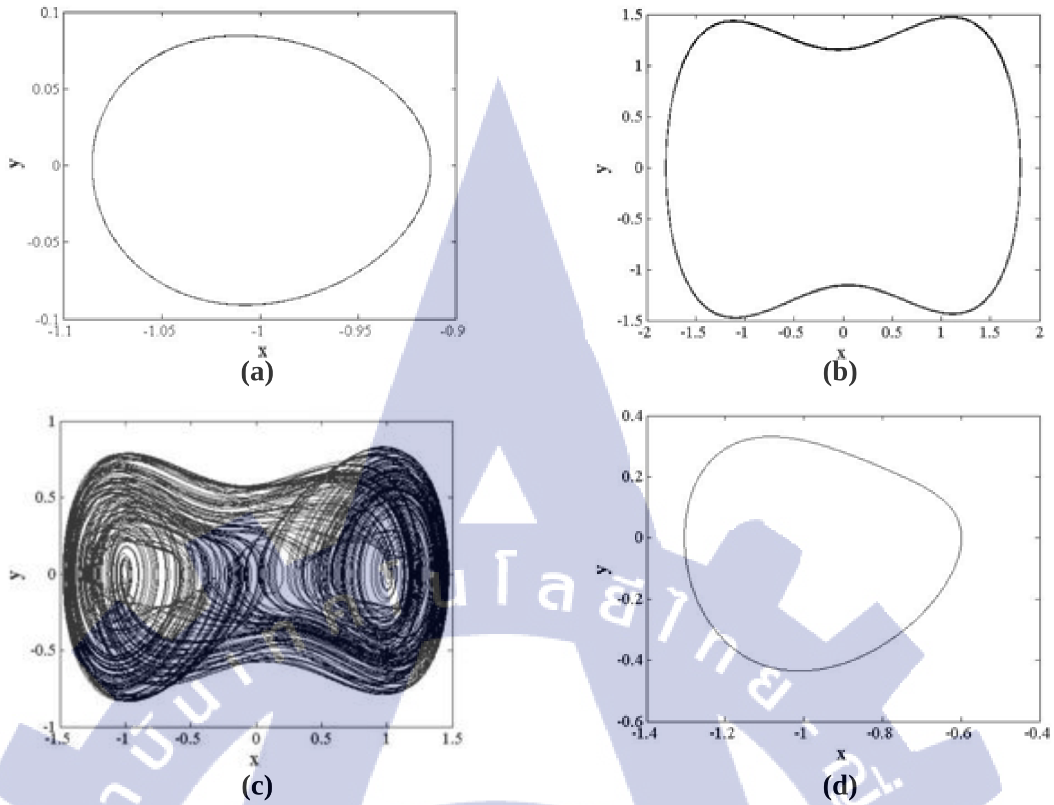


Figure 4.3 The dynamics forming of the Duffing chaotic attractor at γ = 0.37 and (a) δ =0.09(periodic), (b) δ =0.1(periodic), (c) δ =0.34(chaotic) and (d) δ=0.76(periodic

Table 4.1 Summary of characteristics of Lyapunov exponents and the corresponding attractor types, dimensions, and dynamical behaviors.

Lyapunov Exponents			**Attractors**	**Dimension**	**Dynamic Behaviors**
LE_1	**LE_2**	**LE_3**			
-	-	-	Equilibrium Point	0	Static
0	-	-	Limit Cycle	1	Periodic
0	0	-	Attracting 2-Torus	2	Quasi-Periodic
0	0	0	Invariant Torus	1 or 2	Quasi- Periodic
+	0	-	Strange	2 to 3	Chaotic

between two nearby initial conditions that grows exponentially in time when averaged along the trajectory, leading to long-term unpredictability property. The Lyapunov exponents can be employed for the estimation of the rate of entropy production and the fractal dimension commonly known as Kaplan-Yorke dimension D_{KY}, i.e.

$$D_{KY} = j + \frac{\sum_{i=1}^{j} LE_i}{|LE_{j+1}|} = 2 + \frac{LE_1 + LE_2}{|LE_3|} \tag{4.6}$$

where$j=n-1$ and n is the dimension of a dynamical system. As will be seen later, the Duffing chaotic oscillator is a 3-dimenisonal ODE system, and therefore the values of D_{KY} for chaos is $2<D_{KY}<3$. Typically, the D_{KY1} can be obtained through the dynamic properties of Duffing chaotic oscillator described in (4.6) where the stochastic noise function $n(t)$ is excluded.The dynamic analysis is initiated by converting the autonomous system (4.5) into a three-dimensional autonomous system as follows;

$$\begin{aligned} \dot{x} &= \omega y \\ \dot{y} &= \omega(-\delta y + x - x^3 + \gamma \cos(z)) \\ \dot{z} &= \Omega \end{aligned} \tag{4.7}$$

It is shown in Table 4.11that the summarize characteristics of Lyapunov exponents and the corresponding attractor types, dimensions, and dynamical behaviors. It is seen in Table 4.1 that any system containing at least one positive LE is defined to be chaotic. The D_{KY} dimension is conjectured to equal the information dimension of the chaotic attractor. The D_{KY} dimension is fractional and can be described as

$$J_1 = \begin{bmatrix} 0 & \omega & 0 \\ \omega(1-3x^2) & -\omega\delta & -\omega\gamma\sin(z) \\ 0 & 0 & 0 \end{bmatrix} \tag{4.8}$$

In order to find the control parameter δ and γ that offers the maximum values of chaoticity and complexity, Figure 4.4 (a) shows the bifurcation diagram of Z-max versus parameter. Figure 4.4 (b) shows the plot of the LEs versus parameter γ indicates the 2 different regions. The LEs trajectories between the region 0.21 to 0.4shows an unsmooth plotting structure of negative LEs and Positive LEs represented the large chaotic window subsequently to the D_{KY} figure and the bifurcation diagram. However, The periodic state can be denoted by the others two existing regions, 0 to 0.2 and 0.41 to 1 provide the largest Lyapunov exponent at γ = 0.39.Figure 4.4 (c) shows the plots of the D_{KY} versus parameter which specified the maximum value of D_{KY} at 2.44.

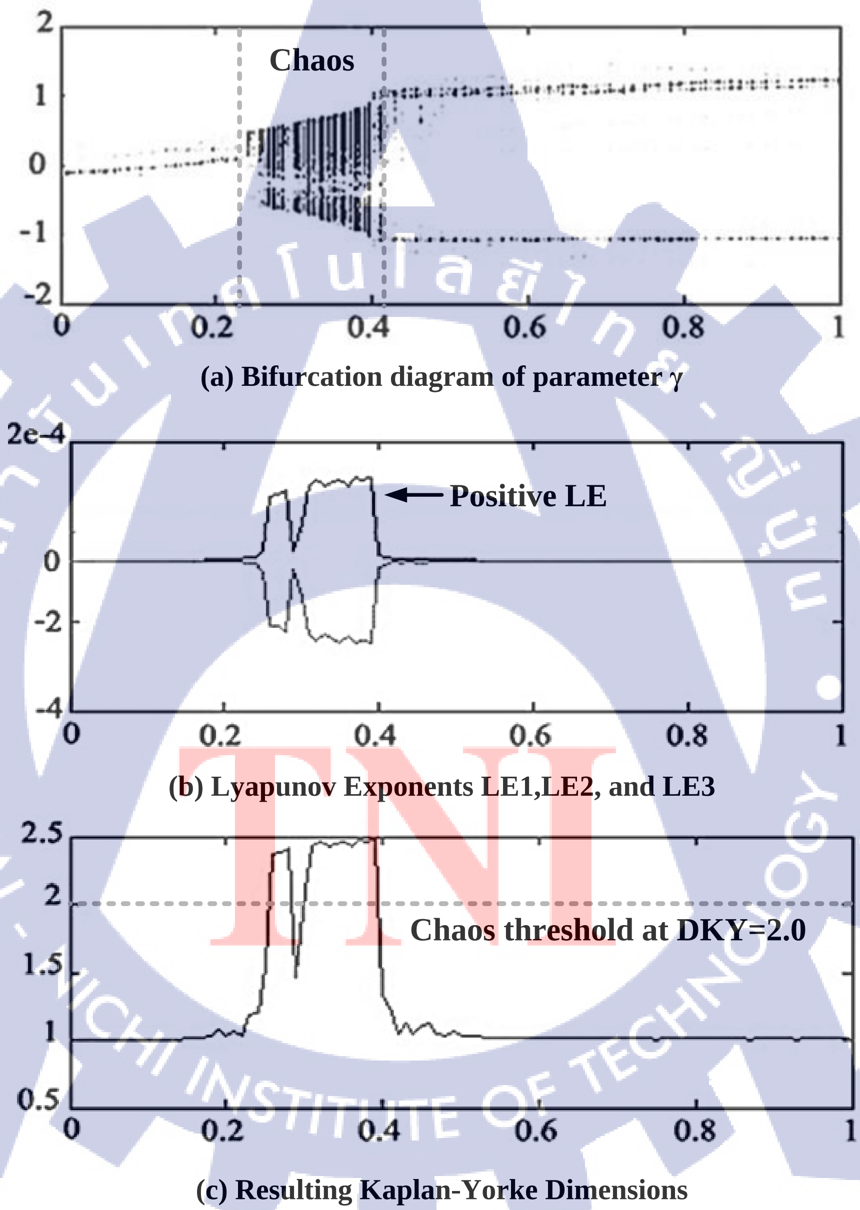


Figure 4.4 Plots of parameter γ versus (a) LEs, (b) D_{KY} and (c) bifurcation diagram where the parameter δ has been fixed at 0.34.

4.1.4 Poincare Section

A Duffing Poincare section is similar to phase space diagram in that it plots points for x versus $\dot{x}$ but accumulated as a function of particular time interval. The time interval typically corresponds to the period of the periodic force. Since sinusoidal signal was primarily sets the frequency ω = 1, Poincare section time interval is 2π. The Poincare step sized equals to $2\pi i$ where i is an integer. As for a primary of investigations, Figure 4.5 shows the principles of period of Poincare section whilst Figure 4.6 shows the Poincare section of Duffing chaotic oscillator at δ =0.21 and γ = 0.39.

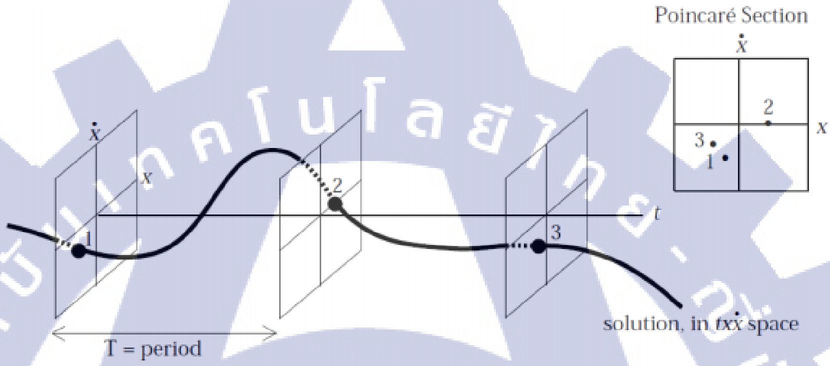


Figure 4.5 The period accumulation of Duffing chaotic Poincare section.

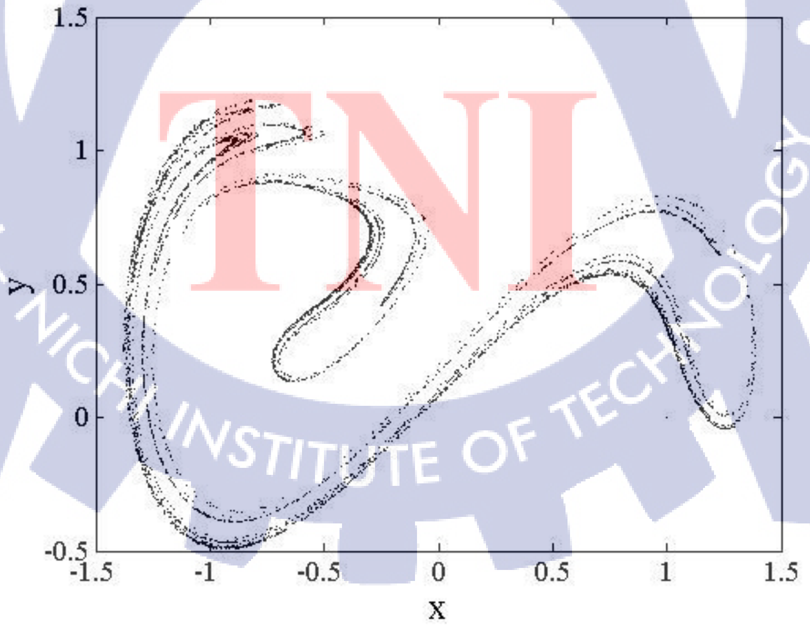


Figure 4.6 The Poincare section of Duffing chaotic oscillator at δ =0.21 and γ = 0.39.

4.2 Proposed Optimizations of Duffing Chaotic Oscillator

4.2.1 Parameter Optimization Techniques and Procedures

It is commonly known that Duffing chaotic equation can exhibit dynamical behaviors, which are characterized by an attractor, a bifurcation diagram, Poincare' section, Lyapunov exponents, and Kaplan York dimension. Such dynamical behaviors involve both periodic and chaotic states, depending on the ranges of equation parameters. In particular for chaotic states, the realization of different parameter values results in distinct chaoticity measured by LEs and complexity measured by D_{KY}. The parameter optimization of equations parameters is therefore necessary in order to obtain the maximum chaoticity and complexity, leading to the appropriate circuit implementation and detecting conditions of WSD systems.Duffing chaotic equation parameters which consequently observe through the sets of chaotic demonstrator including chaotic time-domain waveform, chaotic attractor, and bifurcation and Lyapunov diagram.Second, the D_{KY2} denotes the maximum complexity of the Duffing chaotic equation parameters based on additional forcing terms through the deviation Δγ and Δω based on the equation.

$$\begin{aligned} \dot{x} &= \omega y \\ \dot{y} &= \omega(-\delta y + x - x^3 + \gamma\cos(\omega t) + f_s(t)) \end{aligned} \tag{4.9}$$

The function $f_s(t)$ is detecting sinusoidal signal under noisy conditions, expressed as

$$f_s(t) = \left[(1+\Delta\gamma)\gamma\cos((1+\Delta\omega)\omega + \Phi)\right] + n(t) \tag{4.10}$$

where$\Delta\gamma$ is an amplitude deviation, $\Delta\omega$ a frequency deviation, Φ is a phase angle, and $n(t)$ is an additive Gaussian noise. The proposed method optimizes the parameters δ and γ that characterize dynamic behaviors and bifurcation boundaries over the entire parameter space using a dimensionless D_{KY} as a detection threshold. Figure 4.7 shows the concept of the propose optimization procedure is based on the maximum value of D_{KY}, which indicates the particular sets of the effective equation parameters. The procedure can be separated into two main stages. First, the periodic driving frequency was arbitrarily set at 10 kHz. The capability of WSD method which immense by additional background of noises can easily accumulated by the variation of δ from 0 to 1 in addition to γ with step-size equal 0.1. All pairs of sampling parameters δ and γ yield the corresponding D_{KY1} values in the parameters mapping space. The maximum D_{KY1} denotes the maximum complexity of the existing Duffing chaotic equation which consequently observes through the sets of chaotic demonstrator including chaotic time-domain waveform, chaotic attractor, and bifurcation and Lyapunov diagram.Second, the D_{KY2} denotes the maximum complexity of the Duffing chaotic equation parameters based on additional forcing terms through the parameters deviation Δγ and Δω. The optimization equation resulted at Generalize parameter of Duffing chaotic equation which the maximum complexity corresponding to those parameters. Last, the two DKY values are compared.

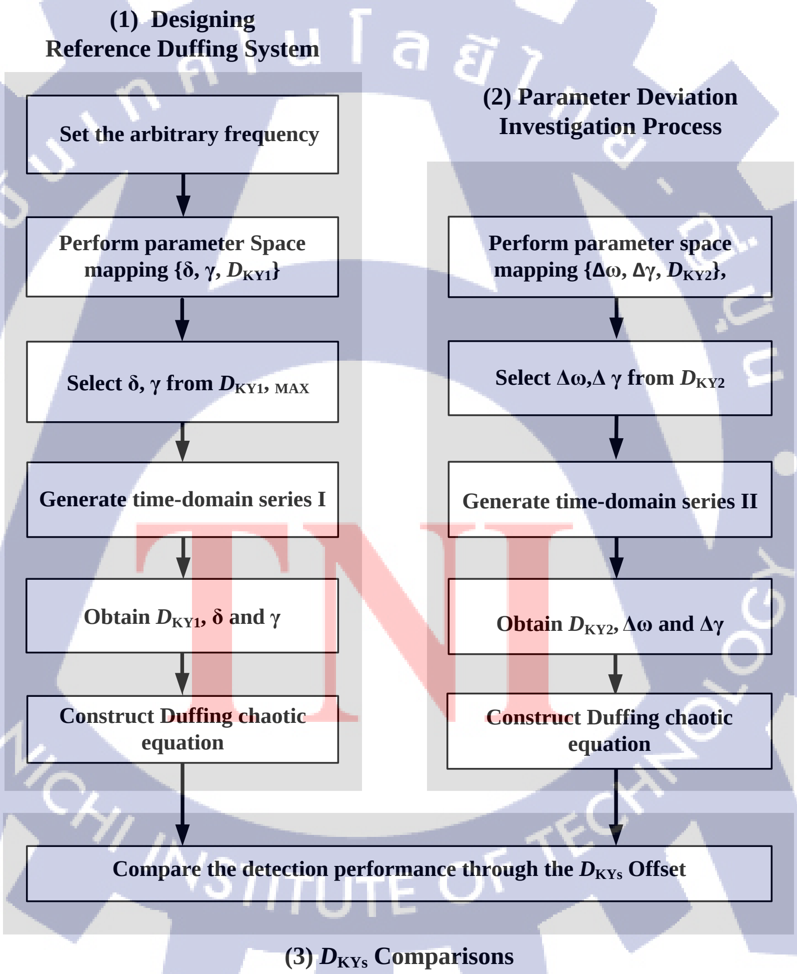


Figure4.7 Block diagram of the optimization procedures.

4.2.2 Parameter Optimization Results

The proposed Duffing chaotic oscillators have been performed in MATLAB and P-spice. The initial conditions of the systems were set to (0,-1.6,-1) and the frequency is set at 10 kHz. The additional forcing terms were neglected from the system due to the zero deviations in both signal amplitude and damping ratio. Table 4.2 summarizes the optimized values of Parameter Setting, Lyapunov Exponents, and Kaplan-Yorke Dimension (D_{KY1}) as the first step in Figure 4.7. Figure 4.8shows the plots of the 3-dimensional parameter mapping space, involving parameters δ, γ, and D_{KY}. As shown in Fig.4.12, the maximum value of D_{KY1} of value 2.44 can be obtained at the coordinate (δ, γ) = (0.34, 0.37), indicating the maximum complexity of the Duffing chaotic system. Therefore, the particular form of the Duffing equation in (4.3) can be expressed with specific values of δ = 0.34 and γ =0.37. In other words,

$$\ddot{x} + 0.34\dot{x} - x + x^3 = 0.37\cos(\omega t) \tag{4.11}$$

Tables 4.3 – 4.6 summarizes Parameter Setting, Lyapunov Exponents, and Kaplan-Yorke Dimension (D_{KY2}) as the second step in Figure 4.7. Figure4.9 shows the detection deviation Versus Parameters deviation of detection.

Table 4.2 Sets of parameters γ and δ with corresponding Values of Lyapunov Exponent and Kaplan-Yorke Dimension, indicating the maximum D_{KY_1}, max.

Parameter Setting		**Lyapunov Exponents (LE^-, LE^0, LE^+)**	**Kaplan-Yorke Dimension(D_{KY1})**
δ	γ		
0.04	0.24	(-0.3802,0,0.1288)	2.32357
0.15	0.47	(-3.0414,0,1.1564)	2.38024
0.25	0.40	(-2.7863,0,0.9697)	2.38170
0.34	0.37	(-2.4943,0,1.1749)	**$D_{KY1,max}$ = 2.47101**
0.41	0.51	(-3.6829,0,1.0942)	2.29711
0.55	0.70	(-4.2887,0,0.8330)	2.19423
0.62	0.89	(-4.8460,0,0.8997)	2.18715
0.74	0.61	(-5.4334,0,0.9028)	2.16168
0.88	0.71	(-6.3520,0,0.8164)	2.12854
0.92	0.75	(-6.5088,0,0.7156)	2.10955

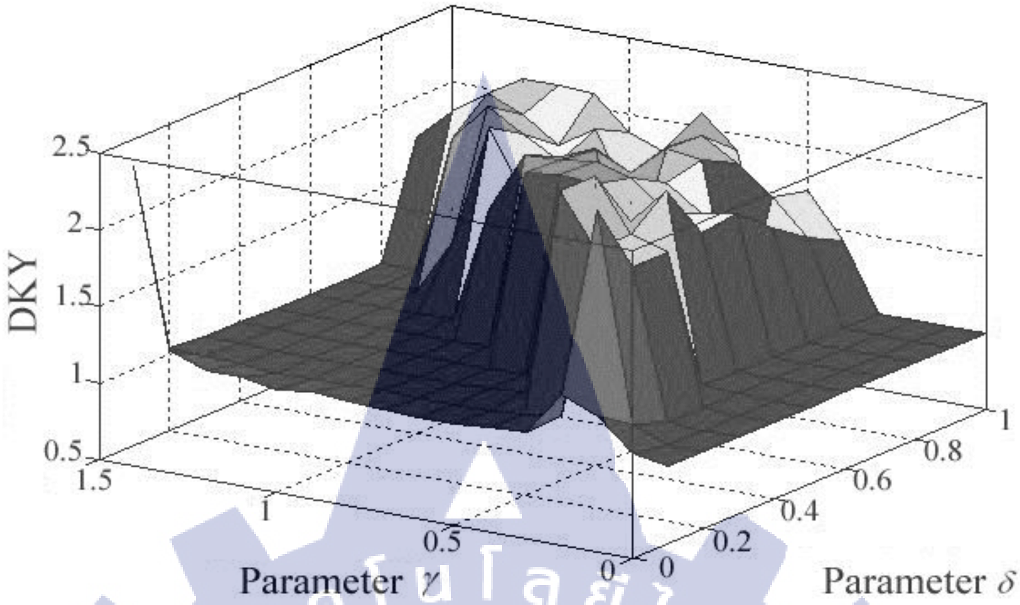


Figure4.8 Plots of the three–dimensional of parameter mapping space.

Table 4.3 Sets of parameters +Δωand+Δγwith the corresponding Values of Lyapunov Exponents and Kaplan-Yorke Dimension, including the Detection deviation between$D_{KY1,MAX}$ and D_{KY2} and its Percentage differences

Parameter Deviation		**LyapunovExponents (LE^+,LE^0,LE^-)**	D_{KY2}	D_{KY1}-D_{KY2}	**%D**
+Δω	*+Δγ*				
0.1	0.0	(-2.6011,0,0.4397)	2.16904	0.30197	12.22
0.2	0.2	(-2.7693,0,0.6079)	2.21952	0.25149	10.17
0.3	0.7	(-2.6906,0,0.5292)	2.19669	0.27432	11.10
0.4	0.2	(-2.8385,0,0.6670)	2.23854	0.23247	9.47
0.5	0.8	(-2.9521,0,0.7907)	2.26785	0.20316	**8.22**
0.6	0.6	(-2.9266,0,0.7652)	2.26146	0.20955	8.48
0.7	0.8	(-2.8482,0,0.6867)	2.24112	0.22989	9.30
0.8	0.0	(-2.5538,0,0.3924)	2.15366	0.31735	12.84
0.9	0.9	(-2.8411,0,0.6798)	2.23926	0.23175	9.378

Table 4.4 Sets of parameters -Δωand+Δγwith the corresponding Values of Lyapunov Exponents and Kaplan-Yorke Dimension, including the detection deviation between$D_{KY1,MAX}$ and D_{KY2} and its Percentage differences.

Parameter Deviation		**LyapunovExponents (LE^+,LE^0,LE^-)**	D_{KY2}	D_{KY1}-D_{KY2}	**%D**
-$\Delta\omega$	**+$\Delta\gamma$**				
-0.1	0.2	(-2.2228,0,0.06143)	2.02764	0.44337	21.87
-0.2	0.0	(-2.5899,0,0.4284)	2.16541	0.30560	14.11
-0.3	0.1	(-2.5779,0,0.4166)	2.16159	0.30942	14.31
-0.4	0.0	(-2.6416,0,0.4802)	2.18177	0.28924	**13.26**
-0.5	0.1	(-1.6132,0,0.5481)	1.66020	0.81081	48.84
-0.6	0.7	(-2.3117,0,0.1503)	2.06503	0.40598	19.66
-0.7	0.3	(-2.1873,0,0.2588)	2.01183	0.45918	22.82
-0.8	0.2	(-2.4965,0,0.3351)	2.13423	0.33678	15.78
-0.9	0.0	(-1.9674,0,0.1940)	1.90139	0.56962	29.96

Table 4.5 Sets of parameters +Δωand-Δγwith the corresponding Values of Lyapunov Exponents and Kaplan-Yorke Dimension, includingthe detection deviation between$D_{KY1,MAX}$and D_{KY2} and its Percentage differences.

Parameter Deviation		**Lyapunov Exponents (LE^+,LE^0,LE^-)**	D_{KY2}	D_{KY1}-D_{KY2}	**%D**
-$\Delta\gamma$	**+$\Delta\omega$**				
-0.1	0.5	(-2.6839,0,0.5225)	2.19469	0.12590	12.59
-0.2	0.9	(-2.7259 ,0,0.5640)	2.20709	0.11958	11.96
-0.3	0.9	(-2.7690 ,0,0.6075)	2.21942	0.11336	11.34
-0.4	0.9	(-2.8426,0,0.6812)	2.23964	0.10330	10.33
-0.5	0.8	(-2.9030,0,0.7416)	2.25545	0.09557	9.557
-0.6	0.9	(-2.9127,0,0.7513)	2.25796	0.09436	9.436
-0.7	0.4	(-2.9263,0,0.7648)	2.26137	0.09270	9.270
-0.8	0.1	(-2.8843,0,0.7229)	2.25064	0.09791	9.791
-0.9	0.5	(-2.9743,0,0.8129)	2.27332	0.08696	**8.696**

Table 4.6 Sets of parameters -Δωand-Δγwith the corresponding Values of Lyapunov Exponents and Kaplan-Yorke Dimension, includingthe Detection deviation between$D_{KY1,MAX}$ and D_{KY2} and its Percentage differences.

Parameter Deviation		**LyapunovExponents (LE^+,LE^0,LE^-)**	D_{KY2}	D_{KY1}-D_{KY2}	**%D**
-Δω	*-Δγ*				
-0.1	-0.4	(-2.6678,0, 0.5065)	2.18984	0.12840	12.84
-0.2	-0.8	(-2.6265,0, 0.4651)	2.17708	0.13501	13.50
-0.3	-0.4	(-2.8568,0, 0.6954)	2.24342	0.10145	10.15
-0.4	-0.6	(-2.9171,0, 0.7558)	2.25907	0.09382	9.38
-0.5	-0.4	(-3.0039,0, 0.8424)	2.28045	0.08356	8.36
-0.6	0.0	(-3.2575,0, 1.0961)	2.33650	0.05757	**5.76**
-0.7	0.0	(-3.1130,0, 0.9517)	2.30570	0.07170	7.17
-0.8	-0.3	(-3.0041,0, 0.8427)	2.28052	0.08353	8.35
-0.9	-0.4	(-2.9218,0, 0.7604)	2.26025	0.09325	9.32

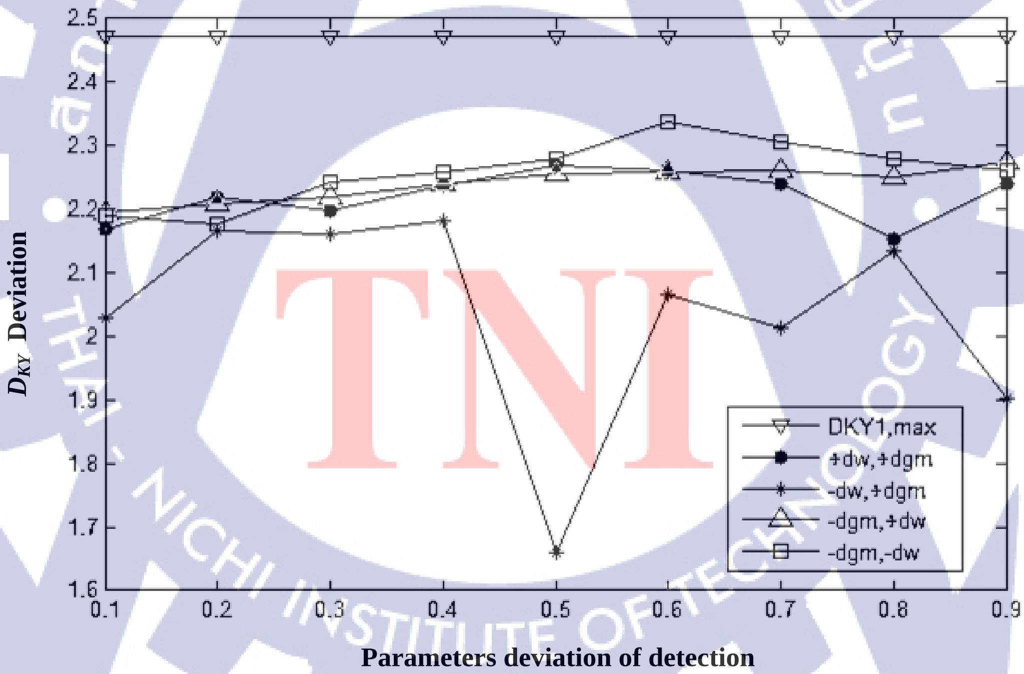


Figure4.9 Detection deviation Versus Parameters deviation of detection

4.3 Proposed Weak Signal Detection using Duffing Oscillator

4.3.1 White Gaussian Noises and its generations

White noise can be viewed as a random signal with a flat power spectral density. In other words, noise is a signal that contains equal power within any frequency band with a fixed width. As for simulation purposes, the command in MALAB is available as *y=awgn*(*x*,*snr*,*sigpower*,*s*) uses *s*, which is a random stream handle, to generate random noise samples with *randn*. If *s* is an integer, then resets the state of *randn* to *s*. The latter usage is obsolete and may be removed in a future release. In Simulink diagram, a repeatable sequence using any Random Number block with the same nonnegative seed and parameters. The seed resets to the specified value each time a simulation starts. By default, the block produces a sequence that has a mean of 0 and a variance of 1. To generate a vector of random numbers with the same mean and variance, specify the Seed parameter as a vector. Figure 4.10 shows white Gaussian noise histograms, time-domain waveforms, and Periodogram power spectral density with variance of 1; (a) Mean = 0.1 and (b) Mean =1.

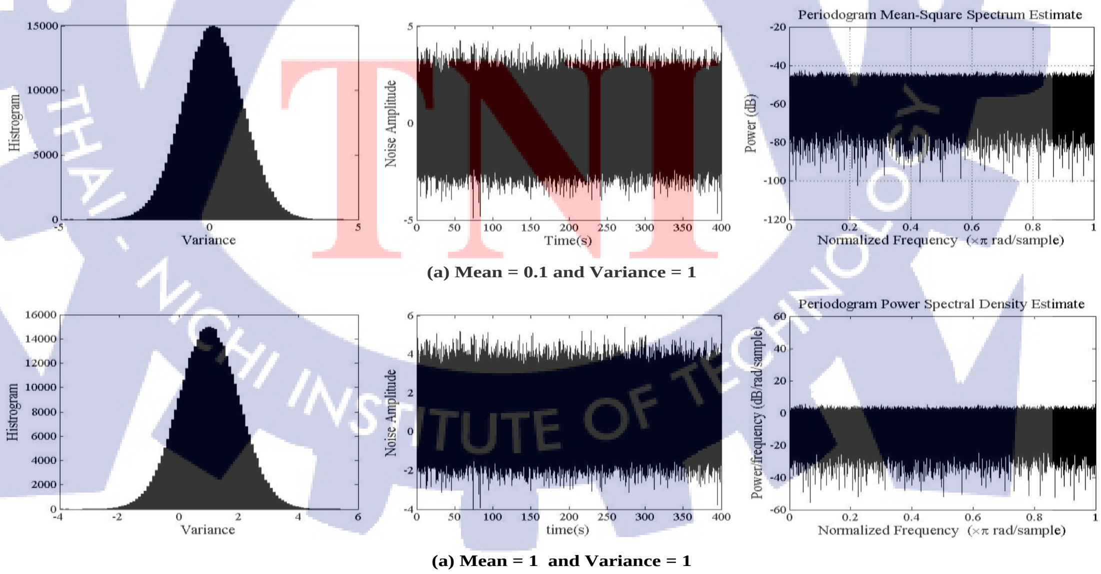


Figure 4.10 White Gaussian noise histograms, time-domain waveforms, and Periodogram power spectral density; (a) Mean = 0.1 and (b) Mean =1.

4.3.2 Proposed Weak Signal Detection Based on Phase-Plane Displays

The proposed technique in this section is to investigate the attractor diagram at different conditions of signals and noises. The method is based on the proposed WSD method that treats the addition of a forcing term defined as $f_s(t)$ into Duffing chaotic oscillator, i.e.

$$\begin{aligned}\dot{x} &= \omega(y) \\ \dot{y} &= \omega(-\delta y + x - x^3 + \gamma_1 \cos(\omega t)) + f_s(t)\end{aligned} \tag{4.12}$$

The function $f_s(t)$ is detecting sinusoidal signal under noisy conditions, expressed as

$$f_s(t) = \gamma_2 \cos(\omega t) + n(t) \tag{4.13}$$

It can be seen that the frequency of the reference signals and the to-be-detected signal are set to be the same while the amplitude are different as summarized in Table 4.3. The weak signal detection. In the detection process, three cases additional signal which is considered as to-be-detected signal are 10mV, 100mV, and 200mV. The variance of noises is considered in two cases including mean values of 0.1 and 1hile the variance is kept at 1.

Figure 4.11 shows Simulink simulation diagram of the Duffing chaotic oscillator in weak signal detection process. Figure 4.12shows the phase diagram of the weak signal detection when additional signal is 10mV while the noise value is 0.1 and 1 and the variance is 1.Figure 4.13shows the phase diagram of the weak signal detection when additional signal is 100mV while the noise value is 0.1 and 1 and the variance is 1.Figure 4.14shows the phase diagram of the weak signal detection when additional signal is 200mV while the noise value is 0.1 and 1 and the variance is 1. It can be considered from the figure that even very small values of the signal to be detected, the phase-diagram can potentially shows the dynamic of Duffing oscillator. However, very large values of noises may disturb the dynamic of Duffing oscillator.

Table 4.7 Summary of parameter used in weak signal detection process

Parameter Conditions	Values	Units
Damping factor (δ)	0.34	-
Nominal Frequency (ω)	1	Rad/s
Reference signal amplitude (γ_1)	370	mV
To-be-detected signal amplitude (γ_2)	varied	mV
Noise signal mean and variance	varied	mV

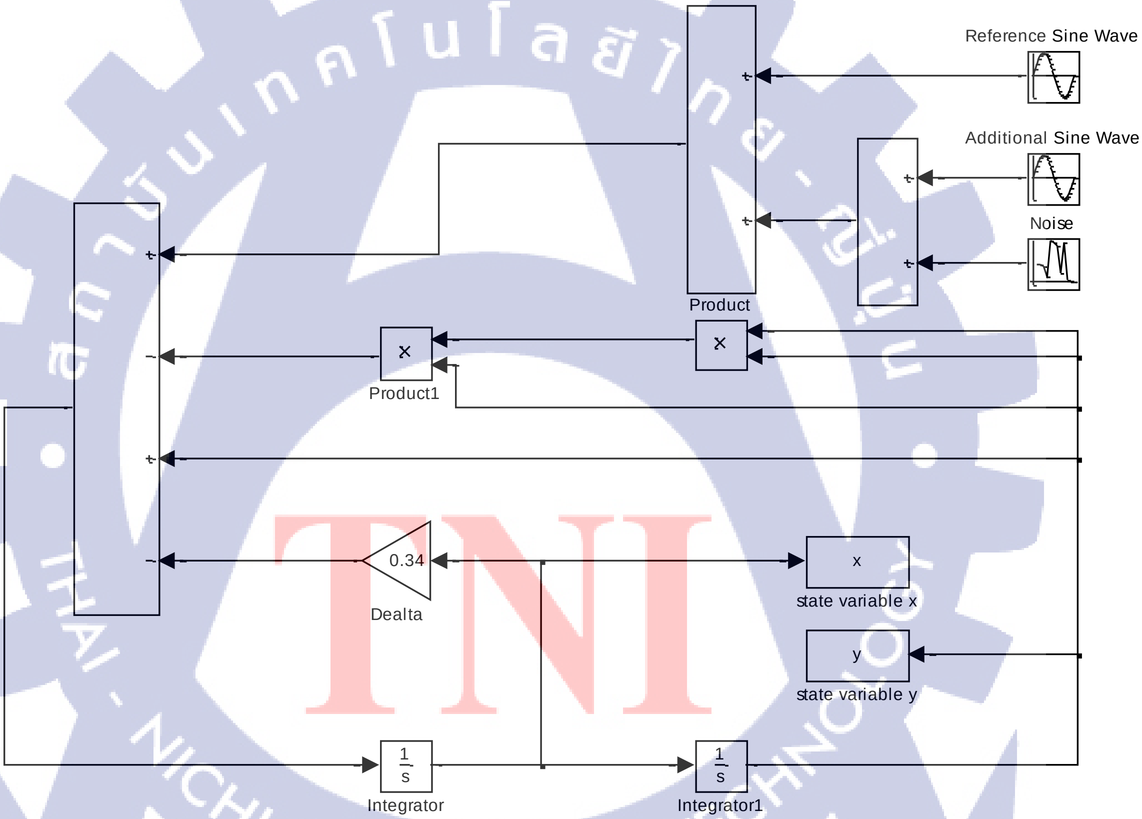
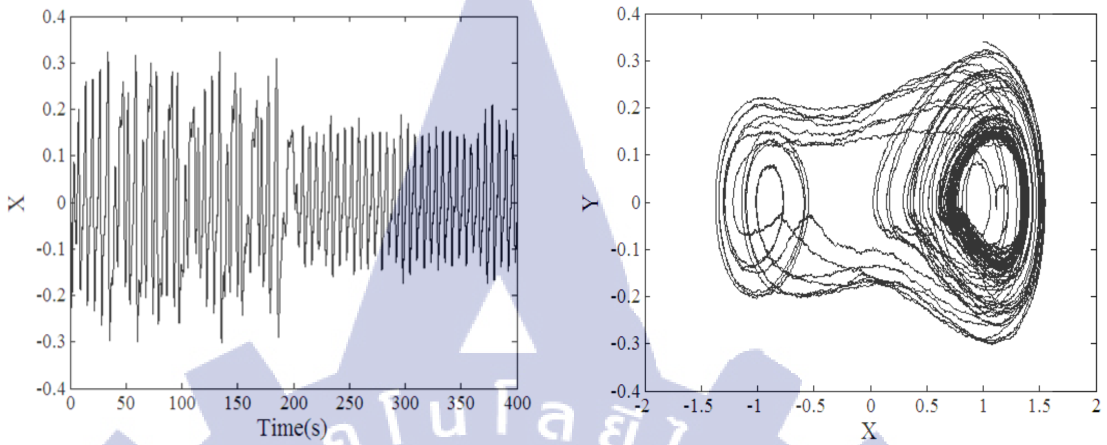
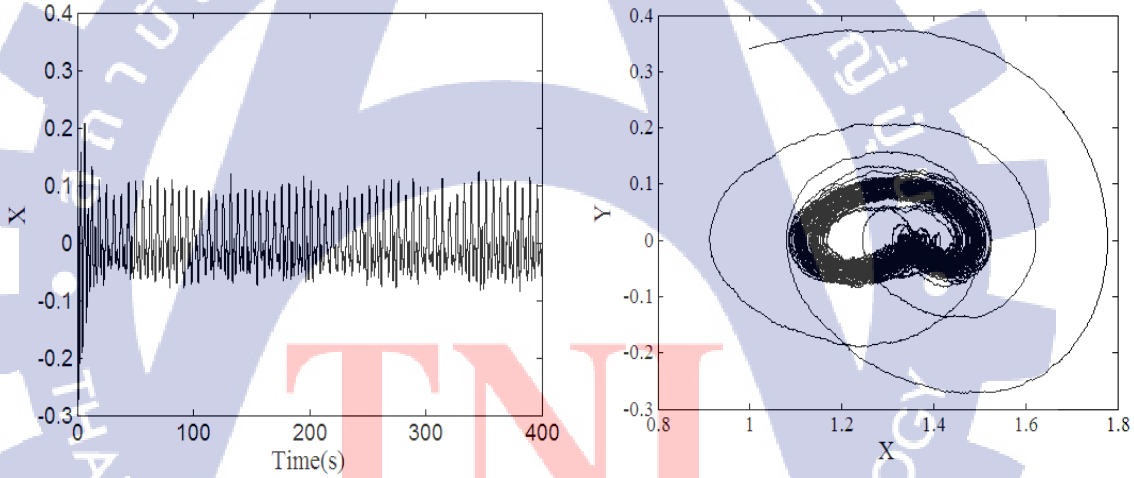


Figure 4.11 Simulink simulation diagram of the Duffing chaotic oscillator in weak signal detection process.

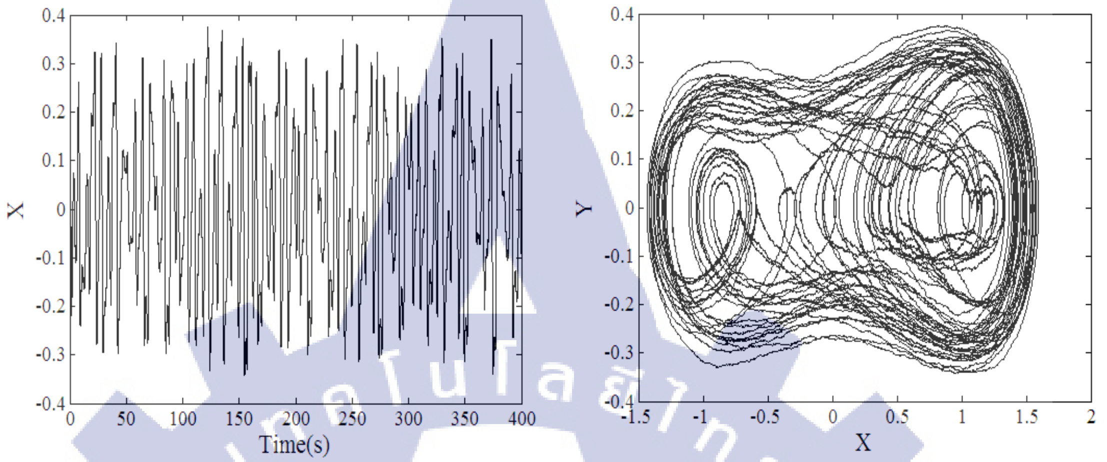


(a) Additional signal at 10mV , Noise mean 0.1 V and Variance 1V

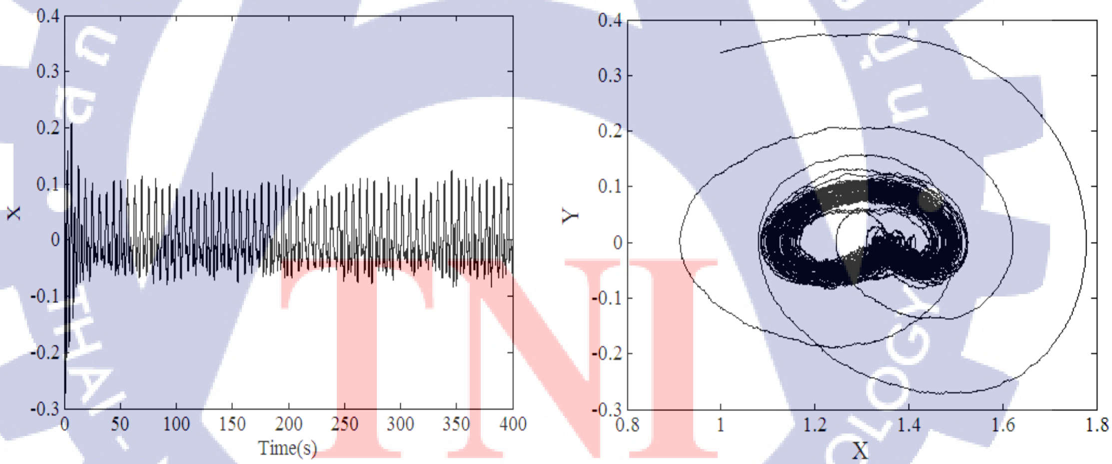


(b) Additional signal at 10mV , Noise mean 1 V and Variance 1V

Figure 4.12 Weak signal detection when additional signal is 10mV while the noise value is 0.1 and 1 and the variance is 1.

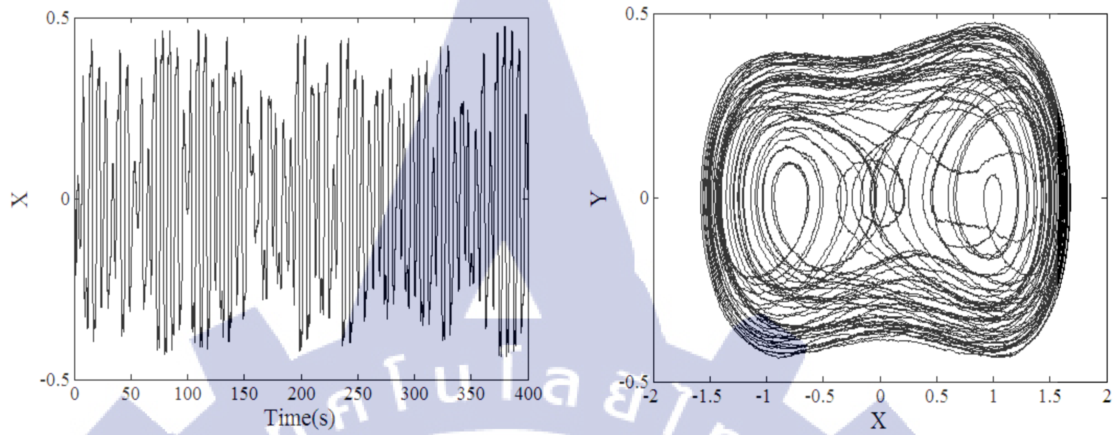


(a) Additional signal at 100mV , Noise mean 0.1 V and Variance 1V

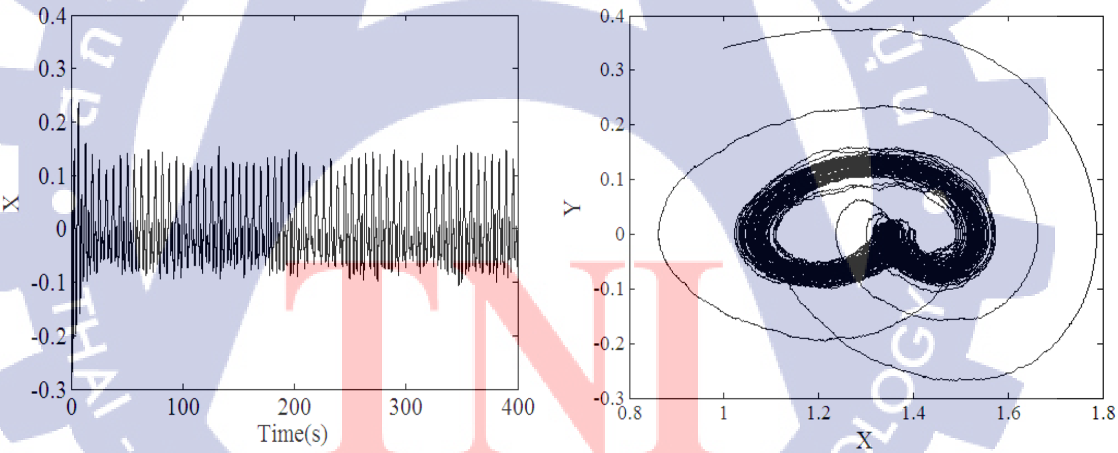


(b) Additional signal at 100mV , Noise mean 1 V and Variance 1V

Figure 4.13 Weak signal detection when additional signal is 100mV while the noise value is 0.1 and 1 and the variance is 1.



(a) Additional signal at 200mV , Noise mean 0.1 V and Variance 1V



(b) Additional signal at 200mV , Noise mean 1 V and Variance 1V

Figure 4.14 Weak signal detection when additional signal is 200mV while the noise value is 0.1 and 1 and the variance is 1.

4.4 Proposed Weak Signal Detection Circuit and Implementations

4.4.1 Circuit Designs

The circuits designed based on the signal correlation principle distinguishes to the weak signal from the noises efficiently. Nonetheless, due to the large circuit lead user to high application cost. The traditional weak signal detection mainly uses the method of linear filtering and signal superimposition to extract the signal. However, it can be seen that such methods often unable to detected signal when the background noise is strong and the detecting signal is weak, cannot meet the needs of weak signal detection requirements. Although modern detection methods have apparently effect, strong adaptability, but most of them have complex structure, difficult to achieve. Therefore, the weak signal detection technology has a new breakthrough when the chaos theory is introduced in the signal detection. The Chaos detection method is different from a variety of existing measuring methods, which is a new signal processing method. The proposed circuits based on Duffing chaotic oscillator has been proposed in this paper. Due to the simplicity, easy to implementation and efficiency computation, Numerical simulation performs through P-spice software in order to model the Duffing chaotic oscillator.

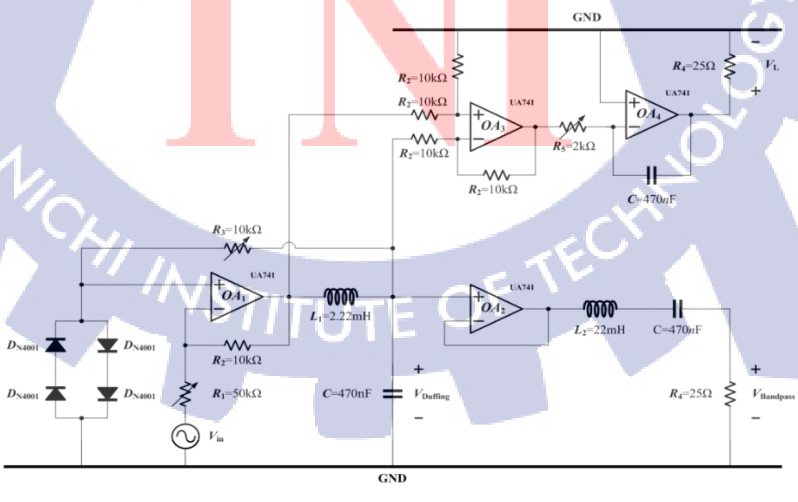


Figure 4.15 Schematic diagram of Duffing Chaotic oscillator with additional selective band-pass filter and Voltage measurement circuit

4.4.2 Circuit Simulation using Pspice

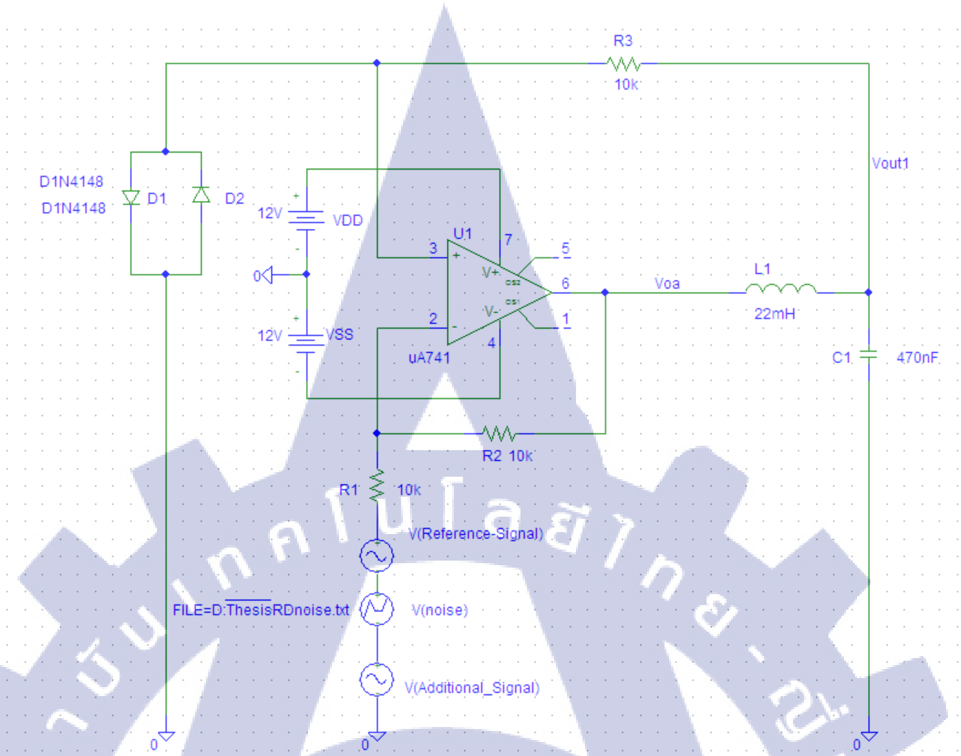


Figure 4.16 Schematic diagram of Duffing Chaotic oscillator in Pspice.

As shown in Figure 4.15. The Circuit can be decomposed into three main parts. First, Duffing chaotic oscillator circuit is driven from RLC oscillator by external periodic signal. Part of circuits is composed with linear component. The nonlinearity components include the feedback loop consisting of resistance and sets of diodes. The operational amplifier performs with two functional processes including buffer for the external sinusoidal signal and amplifying for positive linear feedback stage. Noise can be embedded into Duffing circuits resulted through the chaotic attractor by generated Gaussian white noise from Labview as the external signal modulated with Duffing output voltage.

The simplest band-pass filters are LC-filter consist of inductor and capacitor. Such components can be connected in series or parallel to an existing circuit, the resulting circuits are named as series resonant or parallel resonant circuits, respectively. The additional circuits can be tune to the particular frequency by the LC resonance properties. In both series and parallel added to the circuits have a resonance frequency which neglecting the effect of the resistance in a theoretical examination. The additional components can be connected to the circuits in different ways including parallel-tuned band-pass and series-tuned band-pass as shown in fig1.In series-tuned circuit, both capacitor and inductor have the same current and Voltage across one of the components leads the current by 90o while the other lags the component by 90o therefore the voltage are consequently different with 180o apart. At the resonance frequency Voltage across the series circuit are zero. Hence, the filter circuits perform as a short circuit. Such a circuit property can be widely utilized in a frequency selective application. Such a circuit can be measured by term of either Voltage output or Current. Therefore the Differential amplifier circuits have been employed in the experiment for current output measurement. Using operational amplifier as a device to measure current by connecting one voltage signal onto one input terminal and another voltage signal onto the other input terminal the output voltage will be proportional to the difference between two input signals. Then the differential amplifier amplified the difference between two voltages making the subtractor circuit setting gain equal to 1. Integrate the differential output voltage over the resister to obtain the current output. The early investigation of the Duffing oscillator was done in Pspice. Figure 4.16shows the schematic diagram of Duffing Chaotic oscillator in Pspice.Figure 4.17 shows the noise generated in Pspice.Figures 4.18 and 4.18 show the detected noise in Pspice for the additional signal of 100mV and 200 mV, respectively. The figures reveal that the Duffing chaotic oscillator still sustains its dynamics behaviors through the two-well attractor.

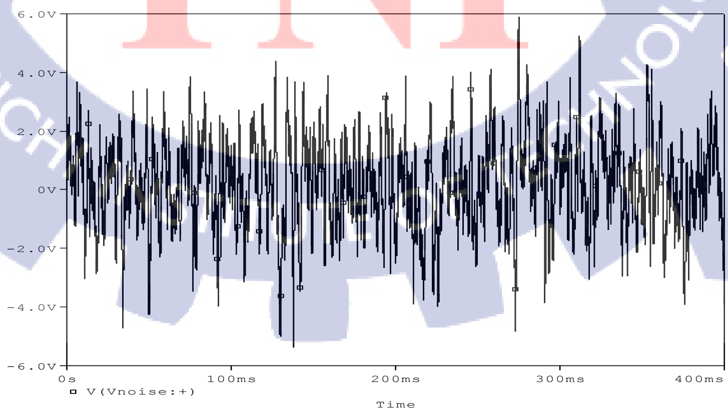
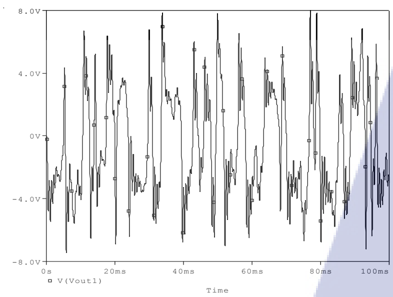
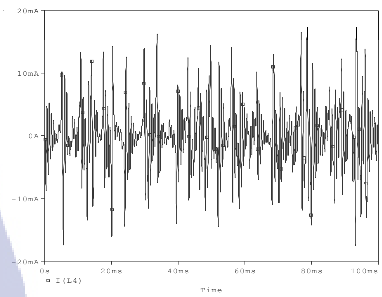


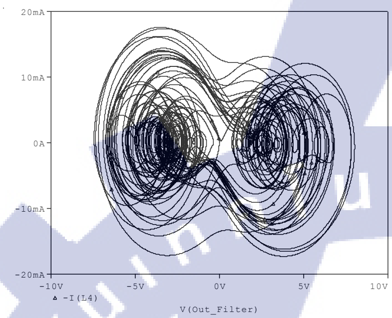
Figure 4.17 Noise generated in Pspice using PWL signal generator.



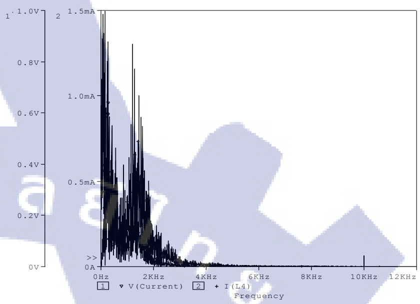
(a) Voltage Waveform V_{out}



(b) Current Waveform I_L

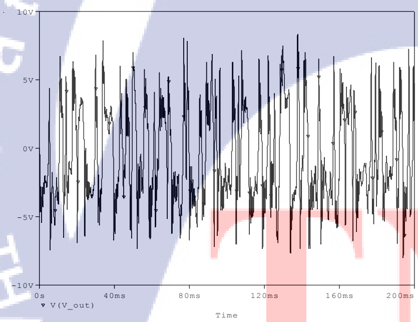


(c) Chaotic Attractor V_{out}-I_L

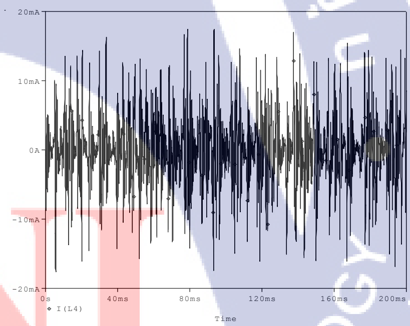


(b) FFT of V_{out} and I_L

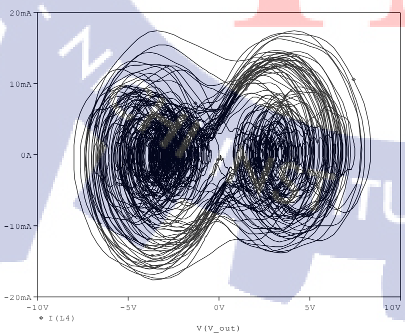
Figure 4.18 Simulations if the case 100mV at frequency of 1.7 kHz at chaoticstate.



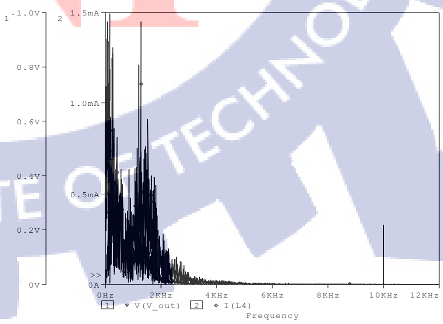
(a) Voltage Waveform V_{out}



(b) Current Waveform I_L



(c) Chaotic Attractor V_{out}-I_L



(b) FFT of V_{out} and I_L

Figure 4.19 Simulations if the case 200 mV at frequency of 1.7 kHz at chaotic state.

4.4.3 Circuit Implementation and Experimental Results

The circuit implementation has been made on board using discrete components. The Op-amp is uA741 with a dual voltage supply of 12V, and the diode is 1N4148. All resistors are 10kΩ. The inductor and capacitor are 22mH and 470nF, respectively.Figure 4.20 shows the circuit implementation of the proposed Duffing chaotic oscillator for use in weak signal detection. It is seen that the circuit is very simple. Chaotic dynamic can be tuned by the potentiometers for the setting of system parameters prior to varying of amplitude or the frequency of the input signals. Figure 4.21 shows the chaotic attractors responding to different frequencies and noise applied. It is seen that different frequency results in different chaotic states including both periodic and chaotic states. Figure 4.22 shows block diagrams of noise generation and signal measurement using LABVIEW. The results of Labview measurements are shown in Figures 4.23-4.25. In particular, Figure 4.26 shows the weak signal detection under noise condition. The attractor still sustains its shape but some noises are added. The input wave form with inherent sinusoidal signal in Figure 4.27 can be detected successfully.

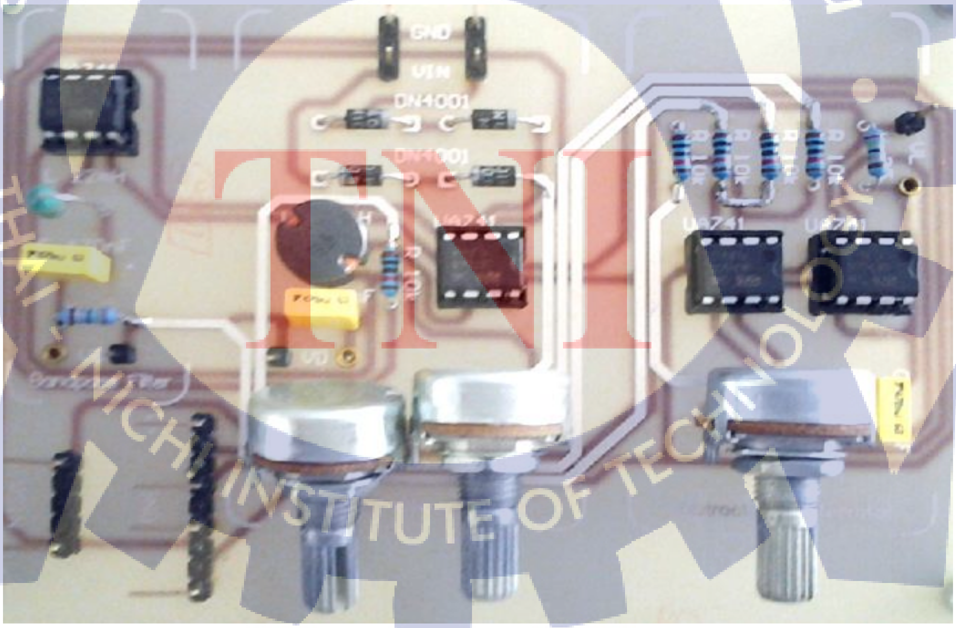
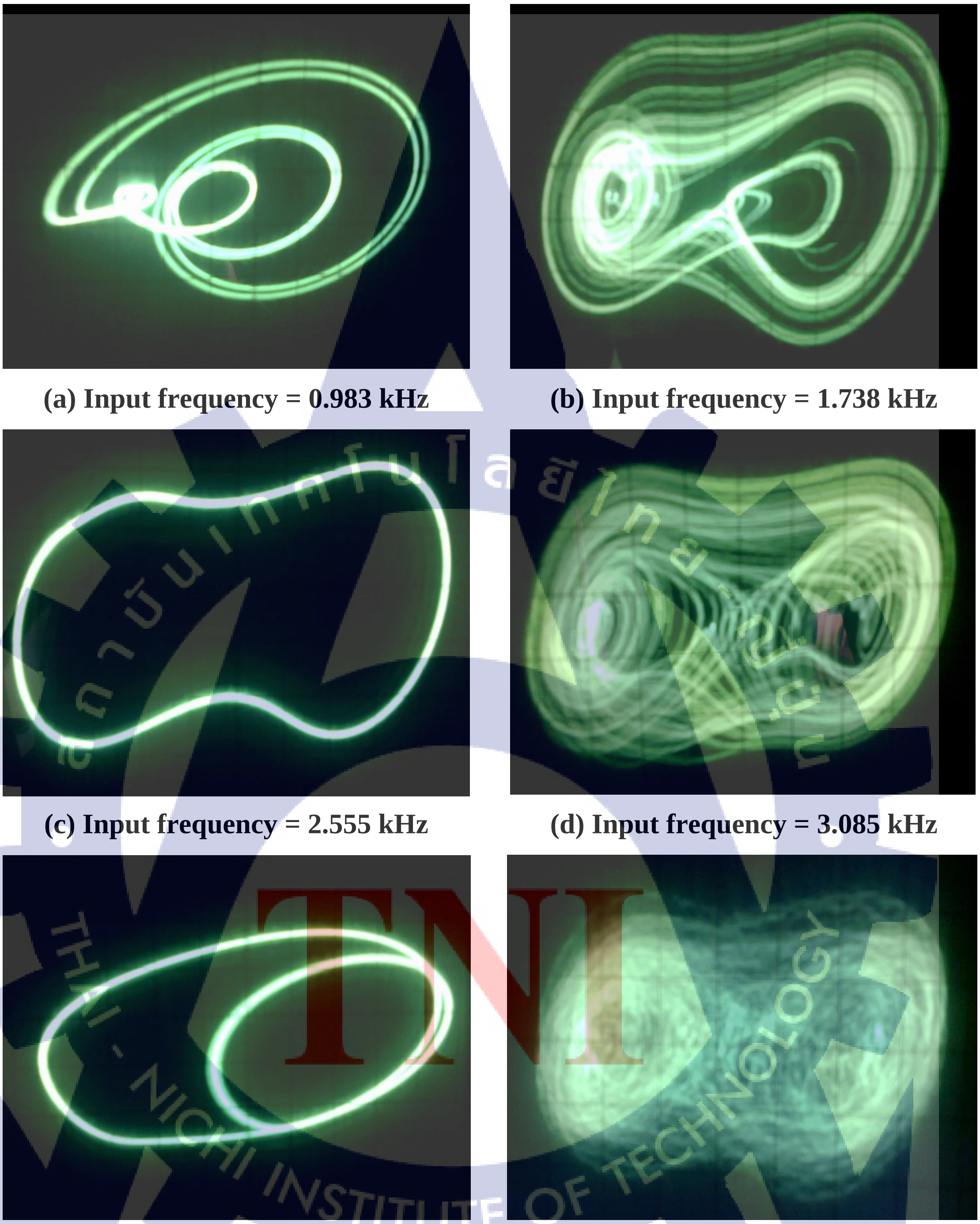


Figure 4.20 Circuit implementation of the proposed Duffing chaotic oscillator for use in weak signal detection.



(a) Input frequency = 0.983 kHz

(b) Input frequency = 1.738 kHz

(c) Input frequency = 2.555 kHz

(d) Input frequency = 3.085 kHz

(c) Input frequency = 3.281 kHz

(d) Input frequency = 3.281 kHz with noise applied

Figure 4.21 Chaotic attractors responding to different frequencies and noise applied.

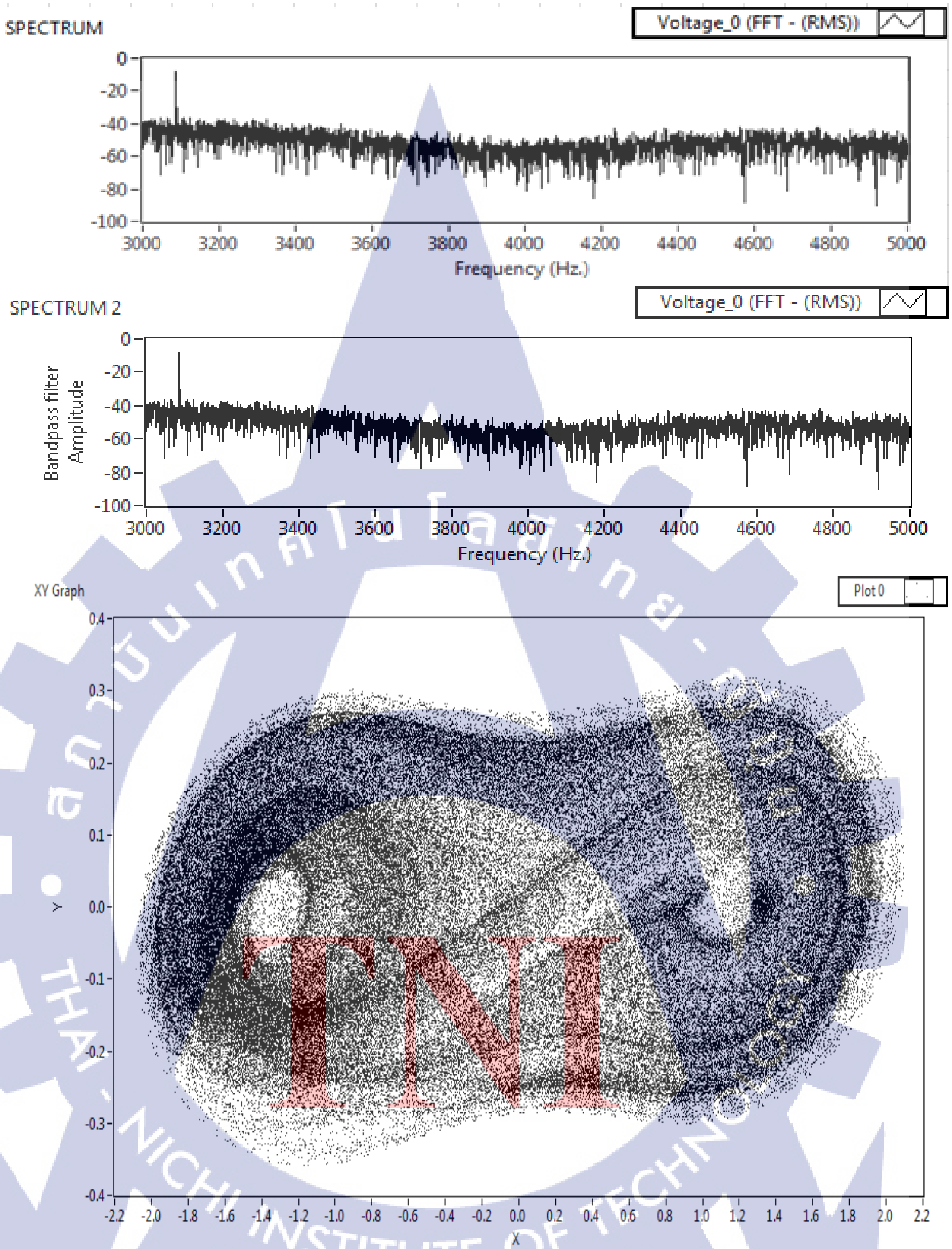


Figure 4.23 Frequency spectrum of Duffing (a) output voltages, (b) band-pass filter at f = 3.085 kHz, and (c) resulting chaotic attractor.

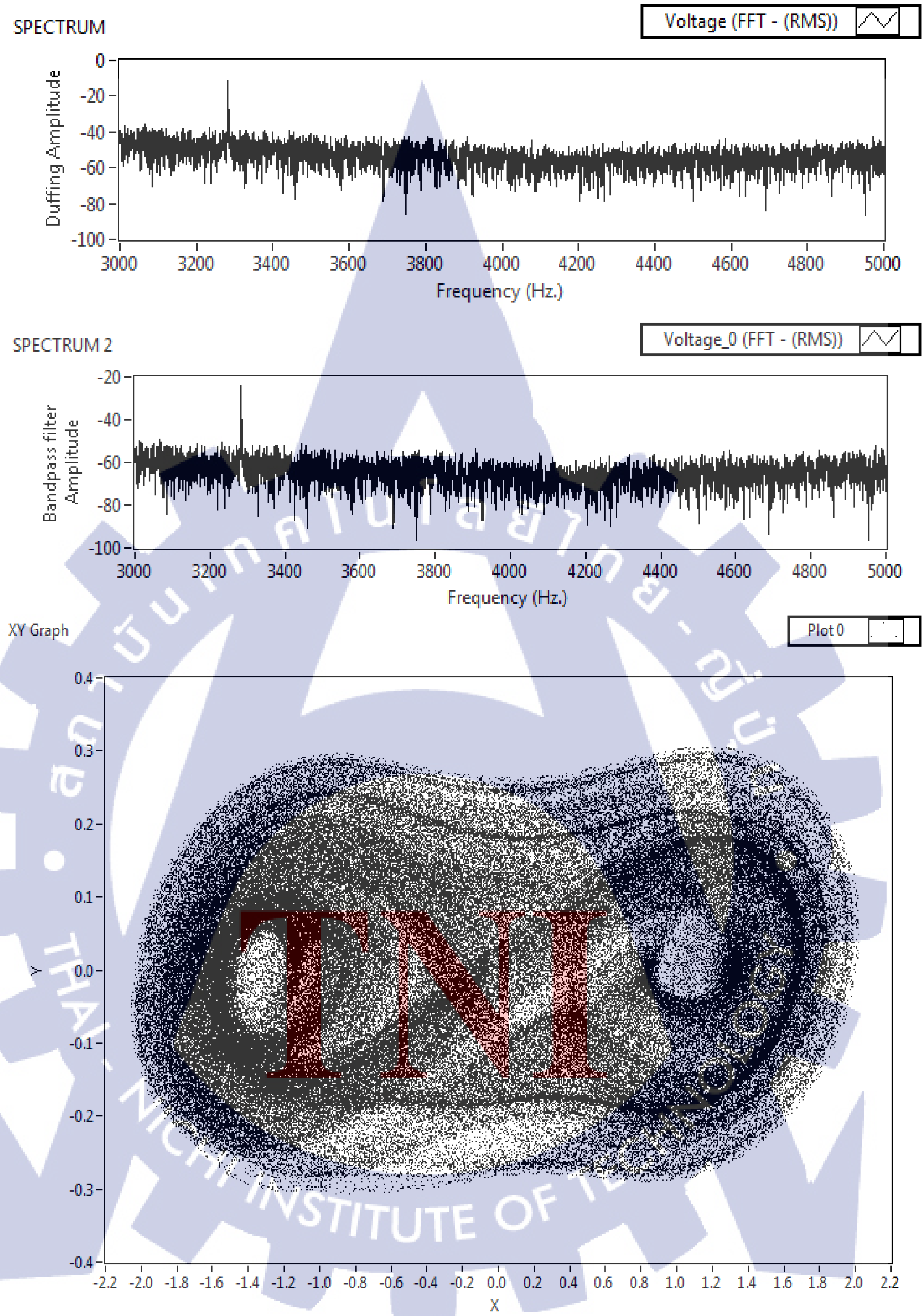


Figure 4.24 Frequency spectrum of Duffing (a) output voltages, (b) band-pass filter at f = 3.281 kHz, and (c) resulting chaotic attractor.

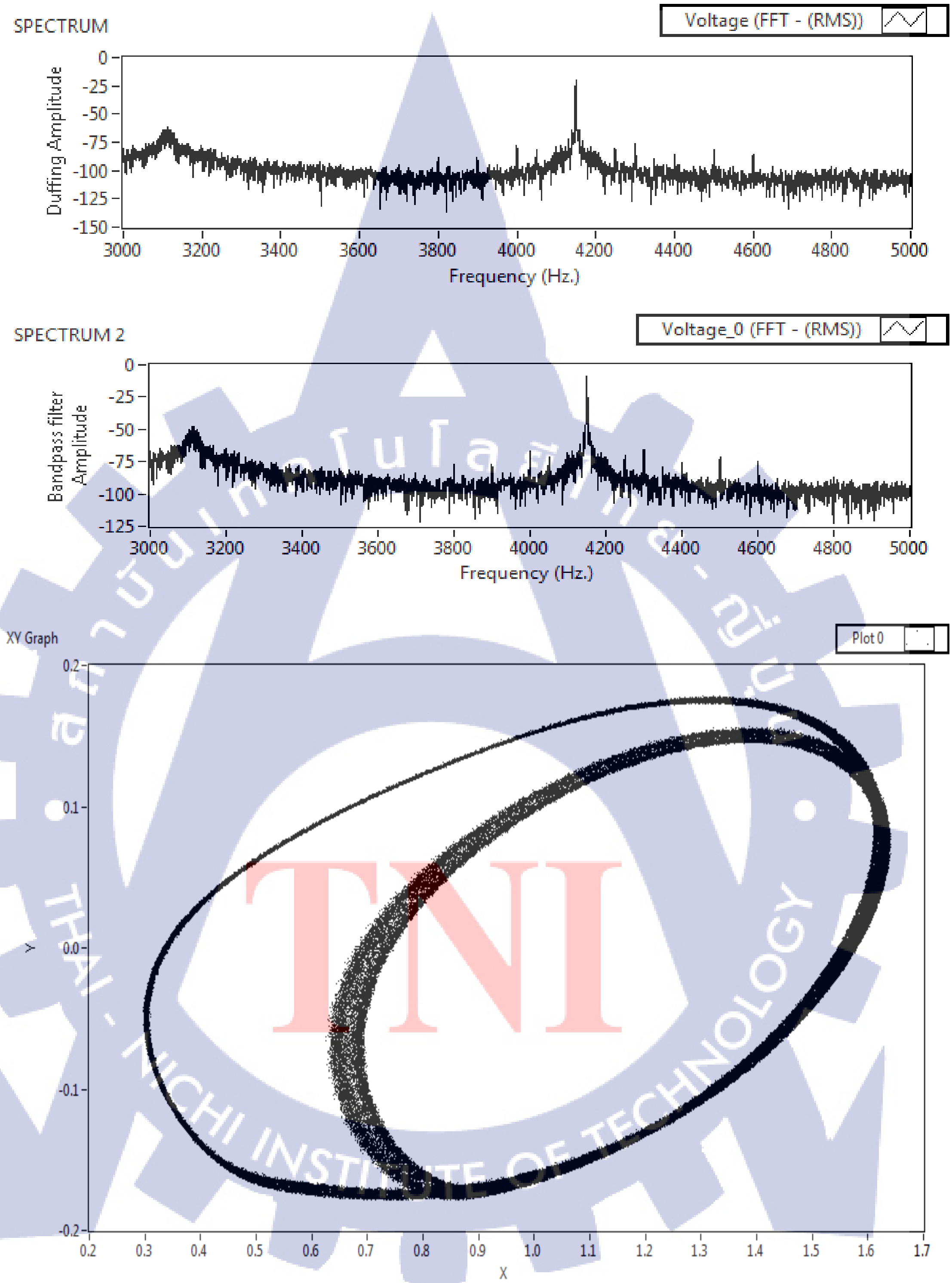


Figure 4.25 Frequency spectrum of Duffing (a) output voltages, (b) band-pass filter at f = 4.14 kHz, and (c) resulting chaotic attractor

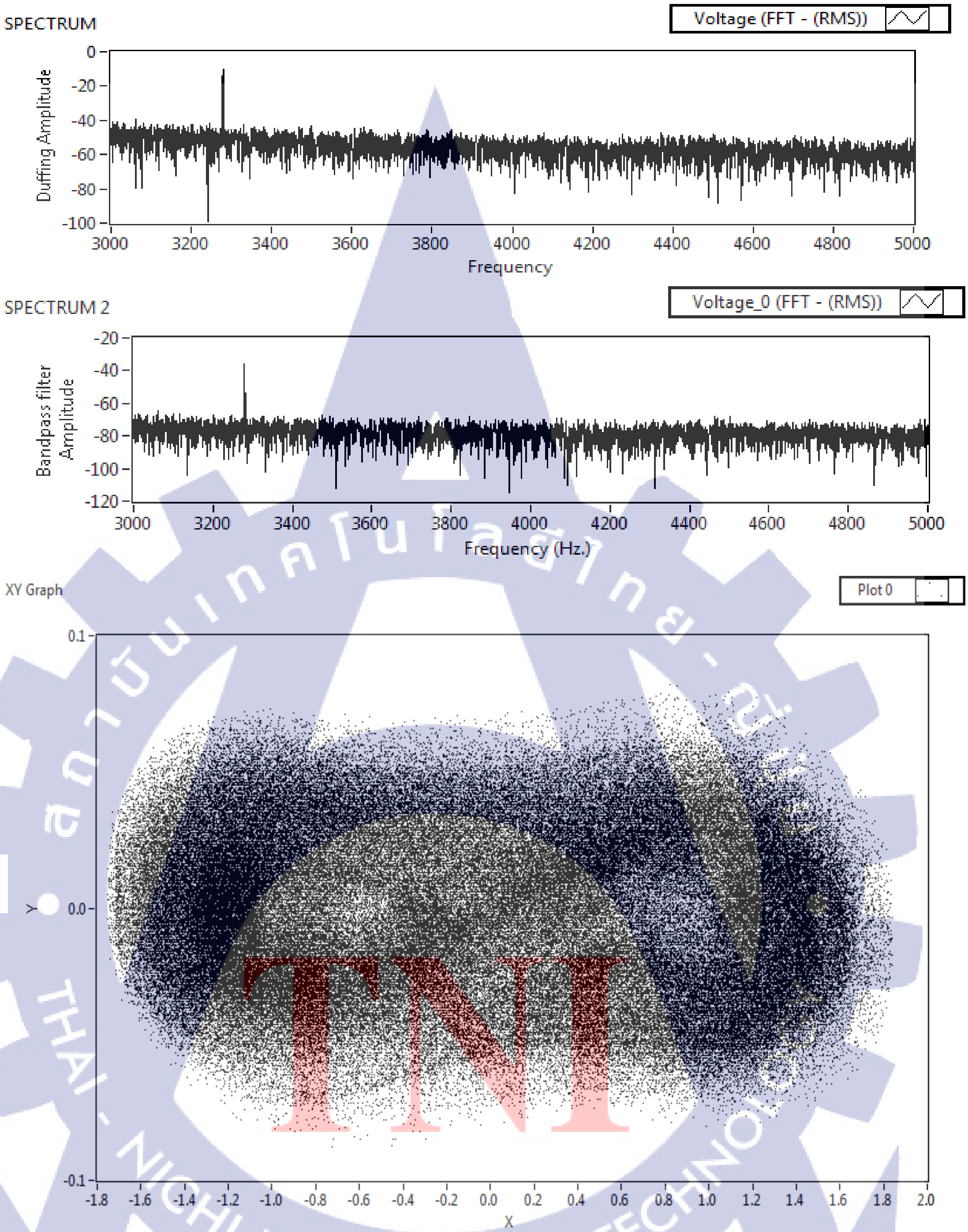
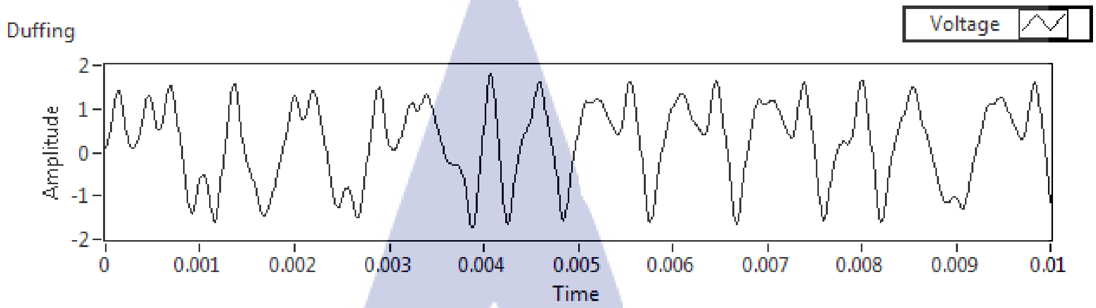
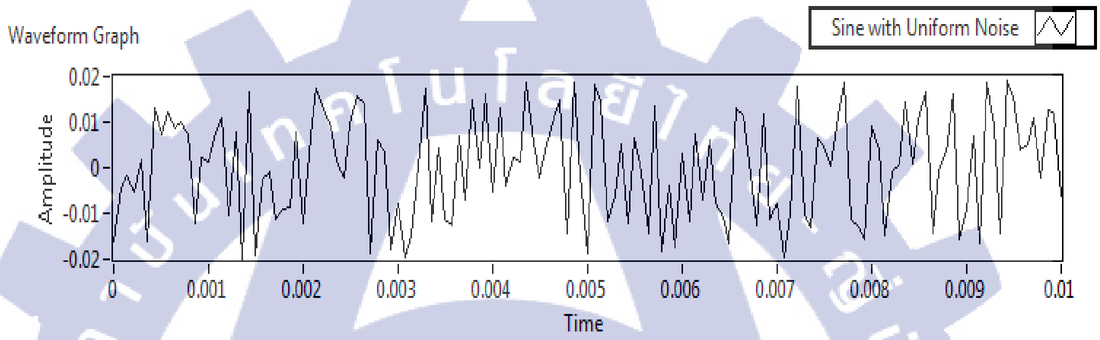


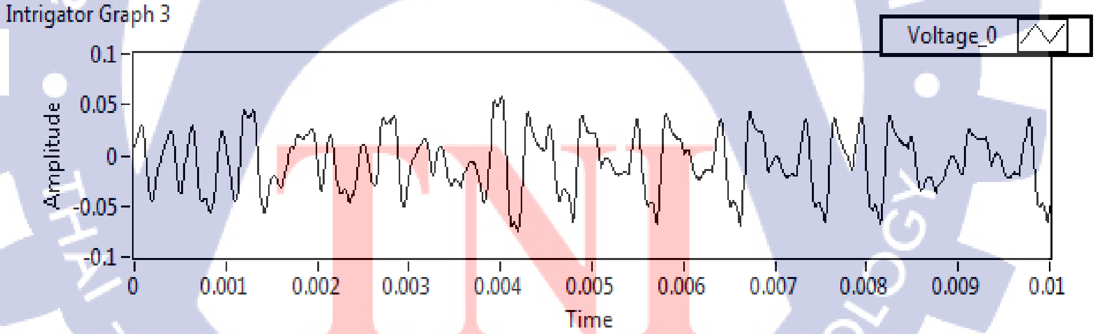
Figure 4.26 Frequency spectrum of Duffing embedded with noises (a) output voltages, (b) band-pass filter at f = 4.14kHz, and (c) resulting chaotic attractor.



(a)



(b)



(c)

Figure 4.27 Particular illustrations of weak signal detection, (a).Duffing chaotic output voltages (b) noise voltage input (c)noise embedded in Duffing output

Chapter 5
Conclusion

5.1 Conclusions

Weak signal detection has been extensively utilized in many signal processing fields such as communications, mechanical industry, medical science and military.Due to the recent high-demand in mechanical industry measurement, weak signal detection has been employed as a tool to detect an Acoustic Emission (AE) in order to exam the corrosion of cutting tools in metal cutting process ,a corrosion in reinforced concrete structures and in the fuel pipes line. Nonetheless, the use of Duffing chaotic oscillators still has some problems. The precision of the system depend on the parameter threshold value still unsolved. Even a small variation of parameter value may cause a dramatically change in the chaotic system behavior. In addition, there is no simple circuit implementation of Duffing oscillator.

This thesis has aimed to perform the mathematical analysis and parameter optimizations of Duffing oscillator, design the weak signal detection methods with the chaotic Duffing oscillator with high accuracy and low SNR, and implement the electronic weak signal detection circuit and system using chaotic oscillator. This thesis has therefore contributed two major significant research outcomes, including parameter optimization and hardware implementation of the Duffing chaotic oscillator. The proposed parameter optimizations aim to achieve the parameter robustness for circuit operation under weak signal detection mode through the comparisons of Kaplan-Yorke conjecture that quantitatively measures the system complexity. The optimized parameters of Duffing equation has been found and employed for the circuit implementation. The circuit implementation of Duffing chaotic oscillator has also been proposed with minimal components with band-pass filtered output and current measurement circuit. The circuit could potentially measure weak signals under noisy conditions through the phase-plane observation in the changes of chaotic attractor.

5.2 Suggestions

As the weak signal detection presented in this thesis is still in an early stage of finding some parameter optimizations and some basic implementation, ones may require some improvements summarized as follows;

5.2.1 The real signals from equipment failures obtained from acoustic emission sensors may be required in future to demonstrate the potentials of the Duffing chaotic oscillator in weak signal detection.

5.2.2 The circuit implementation may be improved as a stand-alone device including battery as a power supply and the connectors for immediate use in real world applications.

5.2.3 It may be useful if the readily available sinusoidal oscillator is integrated into the Duffing chaotic oscillator for use as a reference signals when comparison is needed.

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Appendix

Appendix A – MATLAB Duffing chaotic attractor code

```
function Attractors

Fs = 1e7;

t  = 0: 1/Fs: 0.03;

y0 = [-1; -0.6;0];

global d g w

d = 0.34;

g = 0.72;

w = 2*pi*10000;

[~, y] = ode45(@run_Attractors, t, y0);

f = y(50000:length(y),:);

subplot(1, 1, 1)

plot(f(:,1), f(:,2));

xlabel('x', 'FontSize', 16, 'FontName', 'Cordia New', 'FontWeight', 'bold')

ylabel('y', 'FontSize', 16, 'FontName', 'Cordia New', 'FontWeight', 'bold')

function F = run_Attractors(~, f)

global d g w

x = f(1);

y = f(2);

z = f(3);

F = zeros(size(f));

F(1) = w*(y);

F(2) = w*(-d*y+x-(x^3)+g*sawtooth(z));

F(3) = w;
```

Appendix B – MATLAB Duffing chaotic attractor code

```
function Attractors
Fs = 1e7;
t  = 0: 1/Fs: 0.03;
y0 = [-1; -0.6;0];
global d g w
d = 0.4;
g = 0.36;
w = 2*pi*10000;

[~, y] = ode45(@run_Attractors, t, y0);
f = y(50000:length(y),:);

subplot(1, 1, 1)
plot(f(:,1), f(:,2));
xlabel('x', 'FontSize', 16, 'FontName', 'Cordia New', 'FontWeight', 'bold')
ylabel('y', 'FontSize', 16, 'FontName', 'Cordia New', 'FontWeight', 'bold')

function F = run_Attractors(~, f)
global d g w

x = f(1);
y = f(2);
z = f(3);

F = zeros(size(f));

F(1) = w*(y);
F(2) = w*(-d*y+x-(x^3)+g*cos(z));
F(3) = w;
```

Appendix C – MATLAB Bifurcation code

```
function Bifurcation

clear all

Fs = 1e6;

t  = [0: 1/Fs: 0.05];
```

```
y0 = [-1; -0.6;0];
gmax        = 1;
gmin        = 0;
Sample_Points = 10;
Step_Size=(gmax-gmin)/Sample_Points;
v  = zeros(Sample_Points+1,length(t));
tp = zeros(Sample_Points+1,1);
global d gm w c
d = 0.34;
w=2*pi*10000;
c=1;
%g = 0.36;
for i = 1: 1: Sample_Points+1;
gm = (i-1)*Step_Size+gmin;
   [t, y] = ode45(@ODE, t, y0);
   A = y(:,2)';
          for j = 1: 1: length(A)
h(i, j) = A(1, j);
end
v(i,:) = feval('FindMax', h(i,:));
v(i, 1: 15) = 0;
tp(i) = (i-1)*Step_Size+gmin;
end
%subplot(2, 2, 1)
plot(tp,v,'b.','MarkerSize',4.5)
xlabel('Parameter gm', 'FontSize', 16, 'FontName', 'Cordia New', 'FontWeight', 'bold')
```

```
ylabel(     'gm max', 'FontSize', 16, 'FontName', 'Cordia New', 'FontWeight', 'bold')
%axis([0 2 -2 2]);
function g = FindMax(h)
g=zeros(1,length(h(1,:)));
for k = 2: 1: (length(h(1,:))-1)
if (h(1, k-1)<h(1, k))&&(h(1, k)>h(1, k+1))
g(1, k) = h(1, k);
end
end
function F = ODE(~, f)
global d gm w c
c=1;
x = f(1);
y = f(2);
z = f(3);
F = zeros(size(f));
F(1) = w*y;
F(2) = w*(-d*y+x-(x^3)+gm*cos(z));
F(3) = w*c;
```

Appendix D – MATLAB Lyapunov code

```
function [Texp,Lexp]=lyapunov(n,rhs_ext_fcn,fcn_integrator,tstart,stept,tend,ystart,ioutp);
%   Lyapunov exponent calcullation for ODE-system.
%   Thealogrithm employed in this m-file for determining Lyapunov
%   exponents was proposed in
```

```
%      A. Wolf, J. B. Swift, H. L. Swinney, and J. A. Vastano,
%     "Determining Lyapunov Exponents from a Time Series," Physica D,
%     Vol. 16, pp. 285-317, 1985.
%   For integrating ODE system can be used any MATLAB ODE-suite methods.
% This function is a part of MATDS program - toolbox for dynamical system investigation
%   See:    http://www.math.rsu.ru/mexmat/kvm/matds/
%   Input parameters:
%     n - number of equation
%     rhs_ext_fcn - handle of function with right hand side of extended ODE-system.
%           This function must include RHS of ODE-system coupled with
%           variational equation (n items of linearized systems, see Example).
%     fcn_integrator - handle of ODE integrator function, for example: @ode45
%     tstart - start values of  independent value (time t)
%     stept - step on t-variable for Gram-Schmidt renormalization procedure.
%     tend - finish value of time
%     ystart - start point of trajectory of ODE system.
%     ioutp - step of print to MATLAB main window.ioutp==0 - no print,
%           if ioutp>0 then each ioutp-th point will be print.
%   Output parameters:
%     Texp - time values
%     Lexp - Lyapunov exponents to each time value.
%   Users have to write their own ODE functions for their specified
%   systems and use handle of this function as rhs_ext_fcn - parameter.
%   Example. Lorenz system:
%           dx/dt = sigma*(y - x)     = f1
%           dy/dt = r*x - y - x*z = f2
```

```
%          dz/dt = x*y - b*z     = f3
%   The Jacobian of system:
%      | -sigma  sigma  0 |
%   J = |   r-z    -1   -x |
%      |    y      x   -b |
%   Then, the variational equation has a form:
%   F = J*Y
%   where Y is a square matrix with the same dimension as J.
%   Corresponding m-file:
%      function f=lorenz_ext(t,X)
%       SIGMA = 10; R = 28; BETA = 8/3;
%       x=X(1); y=X(2); z=X(3);
%       Y= [X(4), X(7), X(10);
%           X(5), X(8), X(11);
%           X(6), X(9), X(12)];
%       f=zeros(9,1);
%       f(1)=SIGMA*(y-x); f(2)=-x*z+R*x-y; f(3)=x*y-BETA*z;
%       Jac=[-SIGMA,SIGMA,0; R-z,-1,-x; y, x,-BETA];
%       f(4:12)=Jac*Y;
%   Run Lyapunov exponent calculation:
%   [T,Res]=lyapunov(3,@lorenz_ext,@ode45,0,0.5,200,[0 1 0],10);
%   See files: lorenz_ext, run_lyap.
% -------------------------------------------------------------------
% Copyright (C) 2004, Govorukhin V.N.
% This file is intended for use with MATLAB and was produced for MATDS-program
% http://www.math.rsu.ru/mexmat/kvm/matds/
```

```
% lyapunov.m is free software. lyapunov.m is distributed in the hope that it
% will be useful, but WITHOUT ANY WARRANTY.
%       n=number of nonlinear odes
%       n2=n*(n+1)=total number of odes
n1=n; n2=n1*(n1+1);
%  Number of steps
nit = round((tend-tstart)/stept);
% Memory allocation
y=zeros(n2,1); cum=zeros(n1,1); y0=y;
gsc=cum; znorm=cum;
% Initial values
y(1:n)=ystart(:);
for i=1:n1 y((n1+1)*i)=1.0; end;
t=tstart;
% Main loop
for ITERLYAP=1:nit
% Solutuion of extended ODE system
  [T,Y] = feval(fcn_integrator,rhs_ext_fcn,[t t+stept],y);
  t=t+stept;
  y=Y(size(Y,1),:);
for i=1:n1
for j=1:n1 y0(n1*i+j)=y(n1*j+i); end;
end;
%       construct new orthonormal basis by gram-schmidt
znorm(1)=0.0;
for j=1:n1 znorm(1)=znorm(1)+y0(n1*j+1)^2; end;
```

```
znorm(1)=sqrt(znorm(1));
for j=1:n1 y0(n1*j+1)=y0(n1*j+1)/znorm(1); end;
for j=2:n1
for k=1:(j-1)
gsc(k)=0.0;
for l=1:n1 gsc(k)=gsc(k)+y0(n1*l+j)*y0(n1*l+k); end;
end;
for k=1:n1
for l=1:(j-1)
y0(n1*k+j)=y0(n1*k+j)-gsc(l)*y0(n1*k+l);
end;
end;
znorm(j)=0.0;
for k=1:n1 znorm(j)=znorm(j)+y0(n1*k+j)^2; end;
znorm(j)=sqrt(znorm(j));
for k=1:n1 y0(n1*k+j)=y0(n1*k+j)/znorm(j); end;
end;
%	update running vector magnitudes
for k=1:n1 cum(k)=cum(k)+log(znorm(k)); end;
%	normalize exponent
for k=1:n1
lp(k)=cum(k)/(t-tstart);
end;
% Output modification
if ITERLYAP==1
Lexp=lp;
```

```
Texp=t;
else
Lexp=[Lexp; lp];
Texp=[Texp; t];
end;
 % if (mod(ITERLYAP,ioutp)==0)
 %    fprintf('t=%6.4f',t);
 %    for k=1:n1 fprintf(' %10.6f',lp(k)); end;
 %    fprintf('\n');
 % end;
for i=1:n1
for j=1:n1
y(n1*j+i)=y0(n1*i+j);
end;
end;
end;
```

Appendix E – MATLAB Run-Lyapunov code

```
IC=[-1 -0.6 0];
amax=1;
amin=0;
jmax=10;
del=(amax-amin)/jmax;
globaldgm
```

```
for j = 1: 1: jmax+1
dgm = (j-1)*del+amin
   [T, Res] = lyapunov(3, @JacobiansMatrices, @ode45, 0, 0.00001, 0.05, IC, 10);
   A1 = Res(end,1);
   A2 = Res(end,2);
   A3 = Res(end,3);
A  = 2+((A1)/abs(A3+A2));
L1(j) = A1;
L2(j) = A2;
L3(j) = A3;
L(j) = A;
tp(j) = (j-1)*del+amin;
end
subplot(2, 2, 1)
plot(tp, L);
xlabel(        'Parameter w', 'FontSize', 16, 'FontName', 'Cordia New', 'FontWeight', 'bold');
ylabel('Kaplan-Yorke Dimension', 'FontSize', 16, 'FontName', 'Cordia New', 'FontWeight',
'bold');

subplot(2, 2, 3)
plot(tp, L1, tp, L2, tp, L3);
xlabel(     'Parameter w', 'FontSize', 16, 'FontName', 'Cordia New', 'FontWeight', 'bold');
ylabel('Lyapunov Exponents', 'FontSize', 16, 'FontName', 'Cordia New', 'FontWeight', 'bold');
```

Appendix F – MATLAB Jacobian matrix code

```
function f = JacobiansMatrices(t, X)
```

```
% Values of parameters

global d gm c dwdgm

gm = 0.37;

d = 0.34;

c = 1;

dw = -0.5;

w=2*pi*10000;

x=X(1); y=X(2); z=X(3);

Y= [X(4), X(7), X(10);  X(5), X(8), X(11);   X(6), X(9), X(12)];

f=zeros(9,1);

f(1) = w*y;

f(2) = w*(-d*y+x-(x^3)+gm*cos(z)+((1+dgm)*gm*cos((1+dw)*z)));

f(3) = w*c;

Jac=[          0,       w,          0; w*(1-3*(X(1)^2)),     w*(-d),   w*((-gm)*sin(X(3))-((1+dw)*(1+dgm)*gm*sin((1+dw)*X(3)))); 0,       0,         0]

%Variational equation

f(4:12)=Jac*Y
```

Appendix G– MATLAB Poincare section code

```
clear

deq=inline('[x(2);x(1)-0.21*x(2)-(x(1))^3+0.39*cos(1*t)]','t','x');

options=odeset('RelTol',1e-4,'AbsTol',1e-4);

[t,xx]=ode45(deq,0: (1)*2*pi: (5000)*2*pi,[1,0]);

plot(xx(:,1),xx(:,2),'.','MarkerSize',1)

fsize=15;

xlabel('x','FontSize',fsize)

ylabel('y','FontSize',fsize)

title('Poincare Section of the Duffing System') ;
```